VOLUME X SECOND SERIES NUMBER 4

OCT 1917
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A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

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New Books. 412

PUBLISHED BY
THE AMERICAN PHYSICAL SOCIETY
LANCASTER, PA., AND ITHACA, N. Y.
Entered at the post-office, at Lancaster, Pa., as second class matter

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Annual Subscription, throughout the Postal Union. $6.00
Single Copies, postpaid, $ .60

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THE PHYSICAL REVIEW, ITHACA, N. Y.

Second Series. *October, 1917* *Vol. X., No. 4.*

THE

PHYSICAL REVIEW.

KINETIC THEORY OF RIGID MOLECULES.

By Yoshio Ishida.

Introduction.

It was shown by Boltzmann[1] that the behavior of a monatomic gas may be studied by means of the partial differential equation

$$\frac{\partial f}{\partial t} + \xi \frac{\partial f}{\partial x} + \eta \frac{\partial f}{\partial y} + \zeta \frac{\partial f}{\partial z} + X \frac{\partial f}{\partial \xi} + Y \frac{\partial f}{\partial \eta} + Z \frac{\partial f}{\partial \zeta} = J,$$

when the gas is an ensemble of the same kind of monatomic molecules. The f is the number of molecules per unit cell; the $x, y, z; \xi, \eta, \zeta; X, Y, Z$ are components of three dimensional space, velocity, and acceleration respectively and the J is the rate of change due to encounters. The obvious extension to any number of dimensions is

$$\frac{\partial f}{\partial t} + \Sigma \frac{\partial (f \dot{x}_i)}{\partial x_i} = J,$$

where the x's are any coördinates to specify the system of the individual molecule.

Now the deductions from this equation may be classified into two categories; namely, those which are independent of the form of J, and those which depend upon the nature of J. The hydrodynamic equations can be derived without knowledge of J, provided we admit the existence of such a function. On the other hand the quantitative determinations of the pressure, the viscosity, and the thermoconductivity can not be effected, unless we know something about J.

Since the form of J depends upon what is assumed concerning the nature and frequency of various types of encounters between the molecules it is convenient to classify the coördinates into two groups, according as they are or are not affected by encounters. Let us call the first, the

[1] Boltzmann, Gas Theorie, Vol. I., § 16.

affected coördinates, and the latter the immune coördinates. During encounters, if there is a function of the affected coördinates such that the sum of the function of the coördinates for one molecule and the same function of the coördinates for the other molecule remains unchanged, we shall call such a function an invariant of the encounter. Confining attention to binary encounters, if we have k affected coördinates of one molecule, then the question is to determine $2k$ variables after encounters in terms of $2k$ variables before encounter. If there are r invariants in this special sense in addition to the one purely numerical invariant, and s other general relations (without arbitrary parameters), then the equations of encounter will involve $2k - (r + s)$ parameters. Let Φ_i be the invariants of encounters including $\Phi_0 = 1$, then the equations $d\sigma_I \int \Phi_i J d\sigma_A = 0$; $i = 0 \cdots r$; will be valid and will give $r + 1$ fundamental equations of what may be called generalized hydrodynamics corresponding to the space of the immune coördinates. The $d\sigma_I$ is an element of the immune space and the $d\sigma_A$ is that of the affected space.

To illustrate the notion, let us consider the case of a monatomic gas.[1] The x, y, z are immune coördinates, and ξ, η, ζ are affected coördinates. The number, the three components of translational momentum, and the energy of the system are invariants of encounters, so that $r = 4$. For the parameters of an encounter, we have the longitude and latitude of the point of contact if we adopt the idea of an elastic sphere; the distance from the asymptotic line and the orienting angle if we choose the conception of central forces. Therefore $s = 0$, and consequently there are no additional relations entering into the consideration, and also there are no more invariants. The conservation of the number gives

$$\int J d\sigma_A = 0 \quad \text{(the numerical invariant)}.$$

where $d\sigma_A = d\xi d\eta d\zeta$, which reduces to the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$

where $\rho = \int m f d\sigma_A$, and $\mathbf{V}(V_x, V_y, V_z) = \mathbf{W}(\xi, \eta, \zeta) - \mathbf{U}(U_x, U_y, U_z)$.[2] The $\mathbf{V}$ is the mass velocity and $\mathbf{U}$ is the velocity of agitation.

The conservation of translational momentum gives

$$\int m \mathbf{W} J d\sigma_A = 0,$$

[1] Boltzmann, Gas Theorie, Vol. I. Maxwell, Collected Works, Vols. I. and II. Lorentz, Collected Works, Vol. II. Kirchhoff, Theoretische Physik, Vol. IV. Hilbert, Math. Annalen, Band 72, 1912, p. 562.

[2] In order to save space, Gibb's vector notation is used in this paper whenever it is convenient. The vectors are indicated by clarendon type.

which, when combined with the equation of continuity, reduces to the equation of motion in hydrodynamics,

$$\rho \left\{ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right\} + \nabla \cdot \Lambda - \rho \mathbf{g} = 0,$$

where Λ is the dyadic of stress and g is the vector of acceleration. Finally the conservation of energy gives

$$\int m \frac{\mathbf{W}^2}{2} J d\sigma_A = 0,$$

which reduces, when combined with two preceding, to

$$\rho \left\{ \frac{\partial e}{\partial t} + \mathbf{V} \cdot \nabla e \right\} + \nabla \cdot \mathbf{h} - \nabla \cdot (\Lambda \cdot \mathbf{V}) - \mathbf{V} \cdot (\nabla \cdot \Lambda) = 0,$$

where $\mathbf{h}$ is the flux vector of Fourier conduction of heat and e is the thermal energy per unit mass.

The present paper considers some features of the kinetic theory of gases under the assumption that the molecules are rigid bodies, having no spherical symmetry. The first part will deal with the general hydrodynamical relations. It is evident that we have to consider the orientation and the angular velocity[1] of each individual molecule besides its space coördinates and translational velocity. For invariants, we have three additional equations stating the conservation of moment of momentum. Then the space coördinates x, y, z and the angles φ, ψ, θ are regarded as immune coördinates so that the general idea explained above leads in the first place to a kind of hydrodynamics of six dimensions. The three angles of orientation are then integrated out so as to leave the suitably modified equations of ordinary hydrodynamics, together with an additional vector equation corresponding to the conservation of moment of momentum, which suggests the possibility of the propagation of gyroscopic disturbances besides the sound waves. It will be shown also in the second part that we can specify such binary encounters by five parameters. Consequently we have twelve variables, with seven invariants and five parameters of encounters, thus forming a complete system in the sense that all the independent invariants have been utilized.

The investigations of the specific heat of gases[2] from the standpoint of the equipartion of energy indicate that we can not treat gases like oxygen or hydrogen as monatomic. Thus we have to consider the energy of rotation, which is caused by asymmetry of shape and loading. The idea

[1] Tisserand, Méc. Céleste, Vol. II. Poisson, Méc., Vol. II. Appell, Méc. Rationelle, Vol. III.

[2] Kirchhoff, Theoretische Physik, Vol. IV., page 169. Jeans, Dynamical Theory of Gases, pages 81 and 171. Raleigh, Theory of Sound, Vol. II., page 18.

of considering gas molecules as rigid bodies was initiated by Maxwell,[1] who computed the impulse if two such bodies were to collide. Later various writers[2] carried out the work for some special cases. The second part of this paper will deal with a collision axiom for a more general type of rigid bodies, and some of its consequences. We shall discuss the distribution of translational and angular velocities, especially the equilibrium distribution and its relation to the H-theorem. The distribution function thus deduced will be utilized to compute the external pressures of such gas molecules.

I. HYDRODYNAMICS.

Let x, y, z; ξ, η, ζ be the translational space and velocity coördinates. For orientational coördinates, we can use the Euler angles φ, ψ, θ; and angular velocities $\omega_1, \omega_2, \omega_3$ (see Figs. 1 and 2). The system of moving

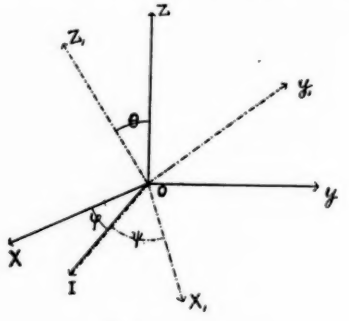


Fig. 1.

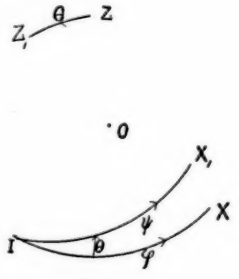


Fig. 2.

axes (x_1, y_1, z_1) and fixed axes (x, y, z) are connected by the following equations (if the translational motion is temporarily neglected).

$$x = lx_1 + my_1 + nz_1$$
$$y = l'x_1 + m'y_1 + n'z_1$$
$$z = l''x_1 + m''y_1 + n''z_1,$$

the nine direction cosines being expressed in terms of the Euler angles, as indicated in the following schema

$$\begin{Bmatrix} l, & m, & n \\ l', & m', & n' \\ l'', & m'', & n'' \end{Bmatrix} = \begin{Bmatrix} \cos\varphi\cos\psi - \sin\varphi\sin\psi\cos\theta, & & \\ \quad -\cos\varphi\sin\psi - \sin\varphi\cos\psi\cos\theta, & \sin\theta\sin\varphi & \\ \sin\varphi\cos\psi + \cos\varphi\sin\psi\cos\theta, & & \\ \quad -\sin\varphi\sin\psi + \cos\varphi\cos\psi\cos\theta, & -\sin\theta\cos\varphi & \\ \sin\psi\sin\theta, & \cos\psi\sin\theta, & \cos\theta \end{Bmatrix}.$$

[1] Maxwell: Collected Works, Vol. I., page 406.

[2] Jeans, Dynamical Theory of Gases, p. 93. Burbury, Phil. Trans., A CLXXXIII., p. 407, 1892. Burnside, Trans. R. S. E., XXXIII., part ii., 1887. N. Delone, Report of Russian Imp. University, 1892.

Then the three components of the angular velocities may be expressed in terms of the time derivatives[1] of the angles,

$$\omega_1 = \dot{\varphi} \sin\theta \sin\psi + \dot{\theta} \cos\psi,$$
$$\omega_2 = \varphi \sin\theta \cos\psi - \dot{\theta} \sin\psi,$$
$$\omega_3 = \dot{\varphi} \cos\theta + \dot{\psi}.$$

The auxiliary formulæ for the change of direction cosines may be obtained directly, thus

$$\dot{l} = m\omega_3 - n\omega_2, \quad \dot{l}' = m'\omega_3 - n'\omega_2, \quad \dot{l}'' = m''\omega_3 - n''\omega_2,$$
$$\dot{m} = n\omega_1 - l\omega_3, \quad \dot{m}' = n'\omega_1 - l'\omega_3, \quad \dot{m}'' = n''\omega_1 - l''\omega_3,$$
$$\dot{n} = l\omega_2 - m\omega_1, \quad \dot{n}' = l'\omega_2 - m'\omega_1, \quad \dot{n}'' = l''\omega_2 - m''\omega_1.$$

If the moving axes are chosen as the principal axes of the body, the dyadic of inertia is

$$\Gamma = Aii + Bjj + Ckk,$$

where A, B, C are the principal moments of inertia, and i, j, k are unit vectors along the moving axes.

Then the differential equations of the motion of a molecule are

$$\dot{x} = \xi, \quad \dot{y} = \eta, \quad \dot{z} = \zeta,$$
$$\dot{\xi} = X, \quad \dot{\eta} = Y, \quad \dot{\zeta} = Z,$$

and

$$\dot{\varphi} = \frac{\sin\psi}{\sin\theta}\omega_1 + \frac{\cos\psi}{\sin\theta}\omega_2,$$
$$\dot{\psi} = \omega_3 - \cot\theta(\sin\psi\omega_1 + \cos\psi\omega_2),$$
$$\dot{\theta} = \cos\psi\omega_1 - \sin\psi\omega_2,$$
$$A\dot{\omega}_1 = (B - C)\omega_2\omega_3 + L,$$
$$B\omega_2 = (C - A)\omega_1\omega_3 + M,$$
$$C\dot{\omega}_3 = (A - B)\omega_1\omega_2 + N,$$

where X, Y, Z are the components of the impressed force, and L, M, N are the components of the impressed couple. The first specify the motion of the center of gravity, and the second specify the rotation of the body referred to the principal axes.

If f is the number of molecules per unit cell in the twelve dimensional region, and J is the rate of the change of this number of molecules due to encounters the Boltzmann equation may be written

$$J = \frac{\partial f}{\partial t} + \Sigma\frac{\partial(f\dot{x})}{\partial x} + \Sigma\frac{\partial(f\dot{\xi})}{\partial \xi} + \Sigma\frac{\partial(f\dot{\varphi})}{\partial \varphi} + \Sigma\frac{\partial(f\dot{\omega}_1)}{\partial \omega_1}.$$

[1] The molecular time derivative is designated by placing a dot above the character, whereas the molar time derivative is designated by the ordinary form d/dt, $\partial/\partial t$.

Let us put for brevity

$$d\tau = dxdydz, \quad d\tau' = d\varphi d\psi d\theta,$$
$$d\sigma = d\xi d\eta d\zeta, \quad d\sigma' = d\omega_1 d\omega_2 d\omega_3.$$

The range of the variables will be

ξ	η	ζ ;	φ	ψ	θ;	ω_1	ω_2	ω_3 .
$-\infty$	$-\infty$	$-\infty$	0	0	0	$-\infty$	$-\infty$	$-\infty$
$+\infty$	$+\infty$	$+\infty$	2π	2π	π	$+\infty$	$+\infty$	$+\infty$

First let us deduce hydrodynamical relations in six dimensions. The conservation of the number gives

$$\int J d\sigma d\sigma' = 0.$$

Now define the density by the equation

$$\rho^* = \int m f d\sigma d\sigma'$$

so that

$$\frac{\partial \rho^*}{\partial t} = \int m \frac{\partial f}{\partial t} d\sigma d\sigma';$$

and also introduce the notation $\mathbf{W} = \mathbf{U} + \mathbf{V}$, where $\mathbf{W}$ is the velocity vector of the center of gravity of the molecule in question, its components being ξ, η, ζ; $\mathbf{V}$ is the vector of mass velocity, and $\mathbf{U}$ the vector of agitation velocity. It follows that $\int m\mathbf{U} f d\sigma d\sigma' = 0$. Then we have

$$m\int J d\sigma d\sigma' = \frac{\partial \rho^*}{\partial t} + \nabla \cdot (\rho^* \mathbf{V}) + \int_{\sigma^1} m \left\{ \Sigma X \int f d\eta d\zeta \right\} d\sigma'$$
$$+ \int m\Sigma \frac{\partial (f\dot{\varphi})}{\partial \varphi} d\sigma d\sigma' + \int_{\sigma} m \left\{ \Sigma \dot{\omega}_1 \int f d\omega_2 d\omega_3 \right\} d\sigma.$$

Further let us specify the nature of the external forces and torques, and the distribution function f in the following manner:

(1) Suppose X, Y, Z are independent of the velocities.

(2) Suppose L, M, N are independent of the angular velocities.

(3) Assume f to be such a function with respect to ξ's and ω's, that the surface integrals become zero as the surfaces extend to infinity.

Then the third and the fifth terms reduce to zero. Let us now write the fourth term as follows

$$\int m\Sigma \frac{\partial (f\dot{\varphi})}{\partial \varphi} d\sigma d\sigma' = \nabla_\phi \cdot (\rho^* \mathbf{N}^*),$$

where

$$\nabla_\phi \cdot = \left(i^* \frac{\partial}{\partial \varphi} + j^* \frac{\partial}{\partial \psi} + k^* \frac{\theta}{\partial \theta} \right).$$

and

$$\rho^*\mathbf{N}^* = \Sigma\int mf\dot{\varphi}d\sigma d\sigma'.$$

Also $\mathbf{N}^*$ is expressible as a linear combination of $\mathbf{M}^*$, where $\mathbf{M}^*$ is the vector of moment of momentum in the fixed space,

$$\left.\begin{aligned} M_1^* &= (ll''A + mm''B + nn''C)N_1^* + nCN_2^* + \cos\varphi N_3^*, \\ M_2^* &= (l'l''A + m'm''B + n'n''C)N_1^* + n'CN_2^* + \sin\varphi N_3^*, \\ M_3^* &= (l''^2A + m''^2B + n''^2C)N_1^* + n''CN_2^* + 0. \end{aligned}\right|$$

The symbol ∇_ϕ may be called angular divergence following the analogy of ordinary space. So finally we have

$$\frac{\partial\rho^*}{\partial t} + \nabla\cdot(\rho^*\mathbf{V}) + \nabla_\phi\cdot(\rho^*\mathbf{N}^*) = 0,$$

which is the equation of continuity in six dimensions.

The conservation of translational momentum, namely $\int m\mathbf{W}Jd\sigma d\sigma'$ may now be considered. If we define the dyadic of stress by

$$\int m\mathbf{U}\mathbf{U}fd\sigma d\sigma' = \Lambda^*,$$

we shall have

$$\int m\mathbf{W}\mathbf{W}\cdot\nabla fd\sigma d\sigma' = \nabla\cdot\{\rho^*\mathbf{V}\mathbf{V} + \Lambda^*\}.$$

Also we have

$$\int m\mathbf{W}\frac{\partial f}{\partial t}d\sigma d\sigma' = \frac{\partial}{\partial t}(\rho^*\mathbf{V}),$$

$$\int m\mathbf{g}\cdot\nabla_\xi fd\sigma d\sigma' = 0,$$

$$\int m\mathbf{W}\mathbf{g}\cdot\nabla_\xi fd\sigma d\sigma' = -\rho^*\mathbf{g}^*,$$

where

$$\nabla_\xi = i\frac{\partial}{\partial\xi} + j\frac{\partial}{\partial\eta} + k\frac{\partial}{\partial\zeta}.$$

With this notation we have

$$\int m\mathbf{W}Jd\sigma d\sigma' = \frac{\partial(\rho^*\mathbf{V})}{\partial t} + \nabla\cdot(p^*\mathbf{V}\mathbf{V} + \Lambda^*) - \rho^*\mathbf{g}^* + \nabla_\phi(\rho^*\mathbf{N}^*)\mathbf{V} = 0.$$

Finally

$$\rho^*\frac{d\mathbf{V}}{dt} + \nabla\cdot\Lambda^* - \rho^*\mathbf{g}^* + \mathbf{V}\nabla_\phi\cdot(\rho^*\mathbf{N}^*) = 0.$$

The conservation of energy may be treated in a similar manner. The energy of translation for the molecule in question is

$$E = \tfrac{1}{2}m(\xi^2 + \eta^2 + \zeta^2),$$

and for the rotational energy we have

$$K = \tfrac{1}{2}(A\omega_1^2 + B\omega_2^2 + C\omega_3^2),$$

so that the conservation of the total energy gives

$$\int (E + K) J d\sigma d\sigma' = 0.$$

Let us adopt the notation

$$\tfrac{1}{2}\int m\mathbf{U}^2 f d\sigma d\sigma' = \rho^* e_t^*,$$

where e_t^* is the thermal energy per unit mass due to the translational velocity,

$$\int K f d\sigma d\sigma' = \rho^* e_r^*$$

where e_r^* is the thermal energy per unit mass due to the angular velocity;

$$e^* = e_t^* + e_r^*;$$

$$\tfrac{1}{2}\int m\mathbf{U}^2\mathbf{U} f d\sigma d\sigma' = \mathbf{h}^*,$$

where $\mathbf{h}^*$ is thermal current density corresponding to the Fourier conduction of heat;

$$\int (\tfrac{1}{2}m\mathbf{U}^2 + K)\Sigma\dot{\varphi} f d\sigma d\sigma' = \mathbf{S}^*,$$

where $\mathbf{S}^*$ is the energy flux carried by the angular velocity;

$$\int f(\omega_1 L + \omega_2 M + \omega_3 N) d\sigma d\sigma' = q^*,$$

where q^* is the work done by the impressed torque. We further have the following reductions:

$$\tfrac{1}{2}\int m\mathbf{W}^2 f d\sigma d\sigma' = \tfrac{1}{2}\rho^*\mathbf{V}^2 + \rho^* e_t^*,$$

$$\tfrac{1}{2}\int m\mathbf{W}^2\mathbf{W} f d\sigma d\sigma' = \mathbf{V}(\tfrac{1}{2}\rho^*\mathbf{V}^2 + \rho^* e_t^*) + \mathbf{V}\cdot\Lambda^* + \mathbf{h}^*.$$

With these auxiliary formulæ we obtain the energy equation

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho^*\mathbf{V}^2 + \rho^* e^*) + \nabla\cdot\{(\tfrac{1}{2}\rho^*\mathbf{V}^2 + \rho^* e^*)\mathbf{V} + \mathbf{V}\cdot\Lambda^* + \mathbf{h}^*\} + \nabla_\phi\cdot\{\tfrac{1}{2}\rho^*\mathbf{V}^2\mathbf{N} + \mathbf{S}^*\} - \mathbf{V}\cdot(\rho^*\mathbf{g}^*) - q^* = 0,$$

which reduces to

$$\rho^*\frac{\partial e^*}{\partial t} + \rho^*\mathbf{V}\cdot\nabla e\cdot + \nabla\cdot(\Lambda^*\cdot\mathbf{V}) + \mathbf{V}\cdot(\nabla\cdot\Lambda^*) + \nabla\cdot\mathbf{h}^* - q^* + \nabla_\phi\cdot\{\tfrac{1}{2}\rho^*\mathbf{V}^2\mathbf{N}^* + \mathbf{S}^*\} = 0.$$

The conservation of angular momentum gives the equations

$$\int \Sigma l A\omega_1 J d\sigma d\sigma' = 0,$$

$$\int \Sigma l' A \omega_1 J d\sigma d\sigma' = 0,$$

$$\int \Sigma l'' A \omega_1 J d\sigma d\sigma' = 0.$$

Let us work out the first component. We have

$$\int \Sigma l A \omega_1 \frac{\partial f}{\partial t} d\sigma d\sigma' = \frac{\partial(\rho^* M_1^*)}{\partial t},$$

$$\int \Sigma l A \omega_1 \nabla \cdot (f\mathbf{W}) d\sigma d\sigma' = \nabla \cdot (\rho^* \mathbf{V} M_1^*),$$

$$\int \Sigma l A \omega_1 \nabla_\xi \cdot (fg) d\sigma d\sigma' = 0.$$

If we write

$$\int f\varphi \Sigma l A \omega_1 d\sigma d\sigma' = H_1^*,$$

where H_1^* is expressible as a linear combination of three components of the rotational energy, we have

$$\int \Sigma l A \omega_1 \Sigma \frac{\partial(f\dot{\varphi})}{\partial \varphi} d\sigma d\sigma' + \int \Sigma l A \omega_1 \Sigma \frac{\partial(f\omega_1)}{\partial \omega_1} d\sigma d\sigma'$$
$$= \nabla_\phi \cdot H_1 - \int f(lA\omega_1 + mB\omega_2 + nC\omega_3) d\sigma d\sigma'.$$

The integral reduces further on account of the Euler equations and the equations of the change of the direction cosines, to

$$-\int f(lL + mM + nN) d\sigma d\sigma' = -\mathbf{G}_1^*.$$

By symmetry we obtain the second and third components, so that we have

$$\frac{\partial(\rho^* \mathbf{M}^*)}{\partial t} + \nabla \cdot (\rho^* \mathbf{V} \mathbf{M}^*) + \nabla_\phi \cdot (H^*) - \mathbf{G}^* = 0.$$

Thus we have deduced a complete set of a kind of hydrodynamic equations for six dimensions.

We can, however, further integrate out the angles of orientation and obtain the resulting system of equations in three dimensions. Since the frame of reference for the Euler angles is arbitrary, the condition that f is a continuous function of φ, ψ, and θ implies

$$[f]_{\phi=0}^{\phi=2\pi} = 0, \quad [f]_{\psi=0}^{\phi=2\pi} = 0, \quad \left[\frac{f}{\sin\theta}\right]_{\theta=0}^{\theta=\pi} = 0.$$

The space of integration $d\sigma_A$ is now $d\tau' d\sigma d\sigma'$ instead of $d\sigma d\sigma'$, and we have to redefine our notations in the following fashion,

$$\rho = \int m f d\tau' d\sigma d\sigma', \quad \text{etc.}$$

It will be seen that, by carrying out the integrations, we have,

for the conservation of number,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0;$$

for the conservation of translational momentum,

$$\frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V}\mathbf{V} + \Lambda) - \rho \mathbf{g} = 0;$$

for the conservation of energy,

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\rho \mathbf{V}^2 + \rho e) + \nabla \cdot \{(\tfrac{1}{2}\rho \mathbf{V}^2 + \rho e)\mathbf{V} + \mathbf{V} \cdot \Lambda + \mathbf{h}\} - q = 0;$$

for the conservation of moment of momentum,

$$\frac{\partial (\rho \mathbf{M})}{\partial t} + \nabla \cdot (\rho \mathbf{V}\mathbf{M}) - \mathbf{G} = 0.$$

We notice at once that these equations are exactly the same as the preceding set provided we assume the angular divergences ∇_ϕ to be zero. We can further simplify the result if we use the Lagrangian time derivatives[1] instead of the Eulerian time derivatives. Thus

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{V} = 0,$$

$$\rho \frac{d\mathbf{V}}{dt} + \nabla \cdot \Lambda - \rho \mathbf{g} = 0,$$

$$\rho \frac{de}{dt} + \nabla \cdot \mathbf{h} + \nabla \cdot (\Lambda \cdot \mathbf{V}) + \mathbf{V} \cdot (\nabla \cdot \Lambda) - q = 0,$$

$$\rho \frac{d\mathbf{M}}{dt} - \mathbf{G} = 0.$$

The first two equations are the same as for the monatomic gas. But the third equation contains the rotational energy as well as the translational, and there is also a contribution of energy due to the work done by the impressed couple. The last equation is the new statement, which suggests that a gas consisting of nonspherical rigid molecules could propagate a kind of gyroscopic disturbance along with compressional waves of the familiar type.

II. Collision Axiom and the Distribution of Velocities.

In the preceding discussion we defined f as the number of molecules per unit cell. This function f will then depend upon thirteen variables

[1] $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla.$

including the time, and our problem is to find these relations. This function f we shall call the distribution function, and assume to be a continuous function with respect to all these variables. Let us call $\rho = \int mfd\tau' d\sigma d\sigma'$ the mass density. The molecular density (say ν) may be absorbed in f, so that we can keep the uniformity of notation. At a given time we can classify all molecules according to twelve properties; then the number of molecules in one of the twelve dimensional cells is

$$fd\tau d\tau' d\sigma d\sigma'.$$

We shall now consider the impact of two molecules which behave like rigid bodies. Let O_1 and O_2 be the two centers of gravity, P the point of impact, and R_1PR_2 the line of impact (normal to the common tangent plane at P) (see Fig. 3). Let the position of P with respect to the principal axes through O_1 be $\mathbf{r}_1$, and the same with respect to those through O_2 be $\mathbf{r}_2$. Let a unit vector along the line of impact with respect to O_1 system be $\mathbf{a}_1$, and the same with respect to O_2 system be $\mathbf{a}_2$. Take for the moments of inertia along the principal axes in these two sets A_1, B_1, C_1; A_2, B_2, C_2 using dyadic notation, then

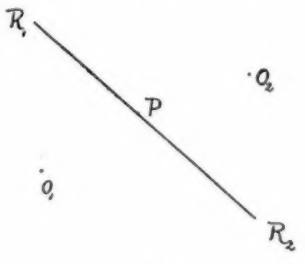


Fig. 3.

$$\Gamma_1 = A_1 i_1 i_1 + B_1 j_1 j_1 + C_1 k_1 k_1,$$

$$\Gamma_2 = A_2 i_2 i_2 + B_2 j_2 j_2 + C_2 k_2 k_2.$$

Take for the mass of the first body m_1, and the second m_2. Let further the translational and the angular velocities of the two bodies before and after impact be

$$\mathbf{W}_1, \mathbf{W}_2, \boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2 \quad \text{and} \quad \overline{\mathbf{W}}_1, \overline{\mathbf{W}}_2, \overline{\boldsymbol{\Omega}}_1, \overline{\boldsymbol{\Omega}}_2,$$

respectively. If we take R for the measure of the impulse due to the impact, we have the following relations;
for the conservation of translational momentum,

$$m_1\overline{\mathbf{W}}_1 = m_1\mathbf{W}_1 + \mathbf{a}_1 R,$$

$$m_2\overline{\mathbf{W}}_2 = m_2\mathbf{W}_2 - \mathbf{a}_2 R;$$

for the conservation of moment of momentum,

$$\Gamma_1 \cdot \overline{\boldsymbol{\Omega}}_1 = \Gamma_1 \cdot \boldsymbol{\Omega}_1 + (\mathbf{r}_1 \times \mathbf{a}_1)R,$$

$$\Gamma_2 \cdot \overline{\boldsymbol{\Omega}}_2 = \Gamma_2 \cdot \boldsymbol{\Omega}_2 - (\mathbf{r}_2 \times \mathbf{a}_2)R;$$

for the conservation of energy,

$$\frac{m_1}{2}\overline{\mathbf{W}}_1^2+\frac{m_2}{2}\overline{\mathbf{W}}_2^2+\tfrac{1}{2}\overline{\Omega}_1\cdot\Gamma_1\cdot\overline{\Omega}_1+\tfrac{1}{2}\overline{\Omega}_2\cdot\Gamma_2\cdot\overline{\Omega}_2$$

$$=\frac{m_1}{2}\mathbf{W}_1^2+\frac{m_2}{2}\mathbf{W}_2^2+\tfrac{1}{2}\Omega_1\cdot\Gamma_1\cdot\Omega_1+\tfrac{1}{2}\Omega_2\cdot\Gamma_2\cdot\Omega_2.$$

From this last equation we can obtain R in terms of the $\mathbf{r}$'s and $\mathbf{a}$'s, substituting the values of the variables before impact for those after impact. Thus

$$R=-2\frac{\mathbf{a}_1\cdot\mathbf{W}_1-\mathbf{a}_2\cdot\mathbf{W}_2+(\mathbf{r}_1\times\mathbf{a}_1)\cdot\Omega_1-(\mathbf{r}_2\times\mathbf{a}_2)\cdot\Omega_2}{\frac{1}{m_1}+\frac{1}{m_2}+[(\mathbf{r}\times\mathbf{a}_1)\cdot\Gamma_1^{-1}\cdot(\mathbf{r}_1\times\mathbf{a}_1)]+[(\mathbf{r}_2\times\mathbf{a}_2)\cdot\Gamma_2^{-1}\cdot(\mathbf{r}_2\times\mathbf{a}_2)]}.$$

If moreover the two molecules are of the same kind, $m_1=m_2$ equal to m say, and $\Gamma_1=\Gamma_2=\Gamma$, then

$$R=-2m\frac{\mathbf{a}_1\cdot(\mathbf{W}_1+\Omega_1\times\mathbf{r}_1)-\mathbf{a}_2\cdot(\mathbf{W}_2+\Omega_2\times\mathbf{r}_2)}{1+m[(\mathbf{r}_1\times\mathbf{a}_1)\cdot\Gamma\cdot(\mathbf{r}_1\times\mathbf{a}_1)+(\mathbf{r}_2\times\mathbf{a}_2)\cdot\Gamma\cdot(\mathbf{r}_2\times\mathbf{a}_2)]}.$$

If we call the direction of the impulse the normal direction (normal to the surfaces), the normal component of the relative velocity of the point of impact will be given by

$$\mathbf{W}_n=\mathbf{a}_1\cdot(\mathbf{W}_1-\mathbf{r}_1\times\Omega_1)-\mathbf{a}_2\cdot(\mathbf{W}_2-\mathbf{r}_2\times\Omega_2).$$

We are now ready to consider the probability of impact of two such molecules. Let us fix our attention only on these two molecules which are going to collide. They will have rotation as well as motion of the center of gravity, and it is necessary for us to observe not only the motion of the centers of gravity but also the behavior of the two points which are going to collide. Let the point of impact of the first body be P and that of the second body be P'. Then if we imagine the first body at rest, P' will describe a curved path before it impinges on P, with such a relative velocity that its normal component may be represented by $\mathbf{W}_n$. Such a path may be found from the differential equations of the motion if we know X, Y, Z and L, M, N.

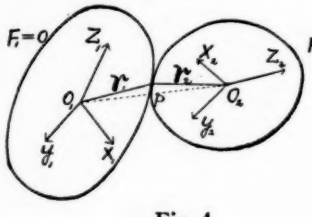


Fig. 4.

As a natural extension of the ordinary supposition in the case of rigid spheres, we shall assume that the probability is proportional to the volume of a cylinder whose base is the element of surface ds at P and whose slant height is the relative velocity of the points of impact.

What we are required to find is, a pair of translational velocities and

a pair of angular velocities after impact in terms of those before impact and the parameters which specify the particular type of impact. One formulation is to take two parameters to specify the point of tangency on the first body and to take the remaining three to specify the orientation of the second body with respect to the first body (see Fig 4). Let $F_1(\mathbf{r}_1) = 0$ and $F_2(\mathbf{r}_2) = 0$ be the two surfaces; then the condition of tangency will give

$$\frac{\partial F_1}{\partial x_1} = \lambda\left(\frac{\partial F_2}{\partial x_2} l_2 + \frac{\partial F_2}{\partial y_2} m_2 + \frac{\partial F_2}{\partial x_2} n_2\right),$$

$$\frac{\partial F_1}{\partial y_1} = \lambda\left(\frac{\partial F_2}{\partial x_2} l_2' + \frac{\partial F_2}{\partial y_2} m_2' + \frac{\partial F_2}{\partial z_2} n_2'\right),$$

$$\frac{\partial F_1}{\partial z_1} = \lambda\left(\frac{\partial F_2}{\partial x_2} l_2'' + \frac{\partial F_2}{\partial y_2} m_2'' + \frac{\partial F_2}{\partial z_2} n_2''\right),$$

where

$$\lambda = \pm \frac{\sqrt{\left(\frac{\partial F_1}{\partial x_1}\right)^2 + \left(\frac{\partial F_1}{\partial y_1}\right)^2 + \left(\frac{\partial F_1}{\partial z_1}\right)^2}}{\sqrt{\left(\frac{\partial F_2}{\partial x_2}\right)^2 + \left(\frac{\partial F_2}{\partial y_2}\right)^2 + \left(\frac{\partial F_2}{\partial z_2}\right)^2}}$$

If the normal is taken in the sense of ∇F, the negative sign is taken. Now the set of the direction cosines l_2, m_2, n_2, etc., may be given by three orienting angles say Φ, Ψ, Θ. Then two parameters on the first body, say the longitude and the latitude, will determine $\partial F_1/\partial x_1$, $\partial F/\partial y_1$, $\partial F_1/\partial z_1$, and consequently $\partial F_2/\partial x$, $\partial F_2/\partial y_2$, $\partial F_2/\partial z_2$ and $\mathbf{r}_2$ may be obtained as functions of these five parameters. Let us designate the element of parametric space (with a proper proportionality factor)by dp; then the probability of impact is $|\mathbf{W}_n| dp d\sigma d\sigma'$. Following the usual method[1] let us conceive two classes of molecules say A and B which are both distributed in the element $d\tau d\tau'$ of space at random. We may suppose the translational velocities and moments of momentum to be uniform so that changes occur only at a collision. Let us classify the encounters into two types α and β, where α designates such encounters that before the collision one of the colliding molecules belongs to the class A and the other to the class B, whereas β designates such encounters that after the collision one of the colliding molecules belongs to the class A and the other to the class B, both types having the same line of impact (the common normal) and the same orientations. The number of collisions of α type in unit time per unit cell of τ and τ' space will be

$$f f' |\mathbf{W}_n| dp d\sigma_1 d\sigma_2 d\sigma_1' d\sigma_2',$$

[1] For instance see Jean's "The Dynamical Theory of Gases," Chap. II.

where $d\sigma_1 d\sigma_1'$ refer to the class A and $d\sigma_2 d\sigma_2'$ to the class B; and f and f' are the distribution functions with arguments having subscripts 1 and 2 respectively. Then the total contribution to the class A due to this α type will be given by integrating the above expression over all possible σ_2 and σ_2', namely

$$d\sigma_1 d\sigma_1' \int\int ff' |\mathbf{W}_n| dp d\sigma_2 d\sigma_2'.$$

The number of collisions of type β in unit time per unit cell of τ and τ' space will be

$$\bar{f}\bar{f}' |\overline{\mathbf{W}}_n| d\bar{p} d\bar{\sigma}_1 d\bar{\sigma}_2 d\bar{\sigma}_1' d\bar{\sigma}_2',$$

where the dashes above the characters express the corresponding functions for the type β, and the total contribution for the class A due to this β type will be, then,

$$d\bar{\sigma}_1 d\bar{\sigma}_1' \int\int \bar{f}\bar{f}' |\overline{\mathbf{W}}_n| d\bar{p} d\bar{\sigma}_2 d\bar{\sigma}_2'.$$

In this theory we assume central symmetry so that $dp = d\bar{p}$. Therefore the number of molecules in the class A is increased by the difference of the two integral expressions above. The difference may be written in the form

$$d\sigma_1 d\sigma_1' \int\int (\bar{f}\bar{f}' - ff') |\mathbf{W}_n| dp d\sigma_2 d\sigma_2'.$$

This involves the fact that the Jacobian[1] of the transformation is equal to unity and $|\overline{\mathbf{W}}_n| = |\mathbf{W}_n|$.[2]

This is the expression for J from this point of view. Thus we formulate the Boltzmann equation as follows:

$$\frac{df}{dt} d\sigma_1 d\sigma_1' = d\sigma_1 d\sigma_1' \left\{ \frac{\partial f}{\partial t} + \Sigma \frac{\partial (f\dot{x})}{\partial x} + \Sigma \frac{\partial (f\dot{\xi})}{\partial \xi} + \Sigma \frac{\partial (f\dot{\varphi})}{\partial \varphi} + \Sigma \frac{\partial (f\dot{\omega}_1)}{\partial \omega} \right\}$$

$$= d\sigma_1 d\sigma_1' \int (\bar{f}\bar{f}' - ff') |W_n| d\sigma_2 d\sigma_2' dp.$$

Let us define

$$S = -k \int f \log f \, d\sigma_1 d\sigma_1',$$

where $S = -kH$, H being Boltzmann's probability function. We obtain in the familiar way

$$\frac{dS}{dt} = \frac{1}{4} k \int (\log \bar{f}\bar{f}' - \log ff')(\bar{f}\bar{f}' - ff') |W_n| d\sigma_1 d\sigma_2 d\sigma_1' d\sigma_2' dp,$$

showing that dS/dt is always positive or zero, and S is an increasing function or else constant. For the steady state S is a maximum and therefore $dS/dt = 0$, so that we have $\bar{f}\bar{f}' - ff' = 0$. This functional

[1] It may be computed easily from the equation of the transformation to be -1, but since we are concerned only with the numerical value the positive sign is taken.

[2] See Maxwell, Collected Works, Vol. I., p. 407.

equation is equivalent to

$$\log \bar{f} + \log \bar{f}' = \log f + \log f',$$

which is the form of an invariant of the encounters. Therefore

$$\log f = \text{an invariant},$$

is a solution, and the complete solution is a linear combination of all invariants. Thus

$$\log f = \alpha_1 N + \alpha_2(m\mathbf{V}^2 + \boldsymbol{\Omega} \cdot \Gamma \cdot \boldsymbol{\Omega}) + \mathbf{b} \cdot \mathbf{V} + \mathbf{c} \cdot \Gamma \cdot \boldsymbol{\Omega},$$

where α_1, α_2; $\mathbf{b}$ and $\mathbf{c}$ are arbitrary constants. Taking the logarithm and rearranging the expression, we have for the distribution function

$$f = \mathcal{A} e^{-\frac{1}{\alpha^2}\left[(\mathbf{V} - \mathbf{V}_0)^2 + \frac{\boldsymbol{\Omega} \cdot \Gamma}{m} \cdot (\boldsymbol{\Omega} - \boldsymbol{\Omega}_0) \right]},$$

and the constants[1] $\mathcal{A}$, α, $\mathbf{V}_0$, $\boldsymbol{\Omega}_0$ are to be determined by the total number, the temperature and the visible motions of translation and rotation.

External Pressure for State of Equilibrium.

We have already found an expression for the impulse, when two rigid bodies impinge on each other. In case of the external pressure, we can simplify the expression, for we can take the plane of the wall as the x-y plane and the axis of z as the direction of the impulse. Thus

$$R = -2m \frac{\mathbf{a} \cdot (\mathbf{W} + \boldsymbol{\Omega} \times \mathbf{r})}{1 + m(\mathbf{r} \times \mathbf{a}) \cdot \Gamma \cdot (\mathbf{r} \times \mathbf{a})},$$

where a has now for its three components l'', m'', n''. It must be noticed that all the vectors in the above expression are referred to the principal axes of the body. The distribution of the orientation being the same as the distribution of the point of tangency o of the x-y plane, we may take the probability of impact to be the product of the normal component of the velocity of the point of contact and the probability of distribution of the z-axis with respect to the center of gravity (see Fig. 5). This latter is given by $1/4\pi \sin\theta \, d\theta d\mu$, where μ is the longitude and θ is the latitude of z on the unit sphere referred to the principal axes. From the geometry of figure we can identify this θ with the previous θ and μ with $\psi + \pi/2$.

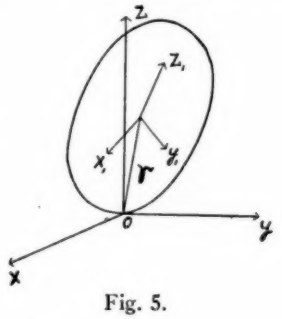


Fig. 5.

[1] These constants may involve x, y, z; ψ, φ, θ; t.

Therefore the probability of impact is

$$|\mathbf{W}_n| \frac{1}{4\pi} \sin\theta d\theta d\psi.$$

Taking the half of this probability because of the assumed central symmetry, we get for the pressure on the x-y plane

$$\frac{1}{8\pi}\int_\sigma\int_{\sigma'}\int_0^{2\pi}\int_0^{\pi} fR|\mathbf{W}_n| d\sigma d\sigma' \sin\theta d\theta d\psi.$$

where

$$\mathbf{W}_n = l''(\xi + \omega_2 Z_1 - \omega_3 y_1)i + m''(\eta + \omega_3 x_1 - \omega_1 z_1)j + n''(\zeta + \omega_1 y_1 - \omega_2 x_1)k,$$

and

$$R = -2m \frac{l''(\xi+\omega_2 z_1-\omega_3 y_1)+m''(\eta+\omega_3 x_1-\omega_1 z_1)+n''(\zeta+\omega_1 y_1-\omega_2 x_1)}{1+m\left[\frac{(y_1 n''-z_1 m'')^2}{A}+\frac{(z_1 l''-x_1 n'')^2}{B}+\frac{(x_1 m''-y_1 l'')^2}{C}\right]}$$

in coördinate expression. Since

$$l'' = \sin\psi\sin\theta, \quad m'' = \cos\psi\sin\theta, \quad n'' = \cos\theta,$$

we have

$$\frac{\partial F}{\partial x_1} = \lambda\sin\psi\sin\theta, \quad \frac{\partial F}{\partial y_1} = \lambda\cos\psi\sin\theta, \quad \frac{\partial F}{\partial z_2} = \lambda\cos\theta,$$

and consequently if we know F, we can solve for x, y, z as functions of ψ and θ. We found above the distribution function f, and since the σ and σ' spaces are independent of the form of F and the orienting angles, we can at once effect the $d\sigma$ and $d\sigma'$ integrations.

If we assume the mass motion and the visible rotation zero, the expression for f may be written

$$f = N\mathcal{A}e^{-(1/\alpha^2)[\xi^2+\eta^2+\zeta^2+(1/m)(A\omega_1^2+B\omega_2^2+C\omega_3^2)]},$$

where $\mathcal{A}$ may be determined by integrating this expression over the whole space, namely

$$N = M\mathcal{A}\int\int\int_{-\infty}^{+\infty}\int\int\int e^{-(1/\alpha^2)\{\xi^2+\eta^2\zeta^2+(1/m)(A\omega_1^2+B\omega_2^2+C\omega_3^2)\}} d\xi d\eta d\zeta d\omega_1 d\omega_2 d\omega_3$$

giving

$$\mathcal{A} = \frac{1}{\alpha^6\pi^3}\frac{\sqrt{ABC}}{m^{\frac{3}{2}}}.$$

In the expression for the pressure, carrying out the integrations with respect to $d\sigma d\sigma'$

$$p = \frac{mN}{8\pi}\int_0^{2\pi}\int_0^{\pi} \alpha^2 \sin\theta\, d\theta d\psi,$$

and if α is independent of the angle

$$p = Nm\frac{\alpha^2}{2}.$$

Putting

$$\frac{\alpha^2}{2} = kT,$$

$$p = NmkT,$$

giving Boyle-Charles's law for this kind of gas.

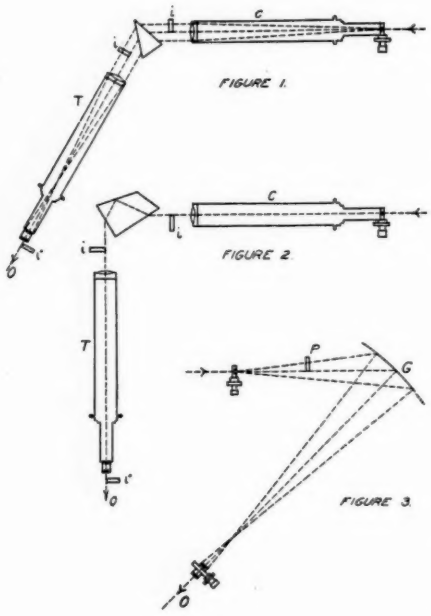
The writer wishes to express his gratitude to Professor A. C. Lunn, who has given suggestions in carrying out this work.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO,
February 2, 1917.

TALBOT'S BANDS AND THE RESOLVING POWER OF SPECTROSCOPES.

BY THOMAS E. DOUBT.

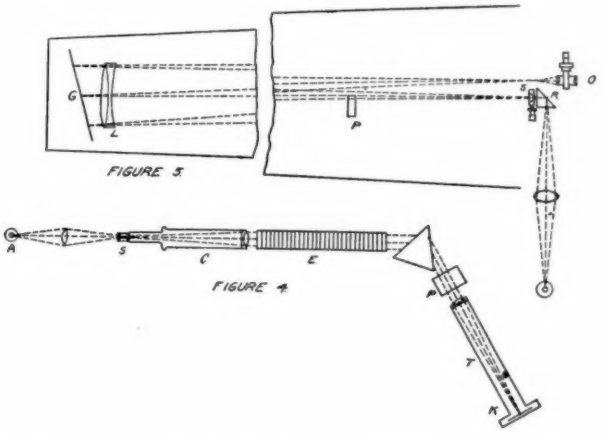
WHEN a pure spectrum formed by a prism is observed visually with one half of the aperture covered with a very thin piece of glass, dark bands may be seen throughout the length of the spectrum crossing it at regular intervals parallel to the slit.[1*] These bands are visible only when the retarding plate is introduced on the side that the blue appears.[2]



The retardation of one half of the beam by the interposed plate may be accomplished in a number of ways.[8] Fig. 1 shows how Talbot's bands may be produced by means of the Bunsen-Kirchhoff spectroscope of single prism. The retarding plate must be introduced at i, i' or i''. In Fig. 2, is shown the method of producing Talbot's bands by means of the Hilger wave-length constant deviation spectroscope.[15] In Fig. 3 is shown the plan for the most convenient arrangement with a Rowland concave grating.[16] The retarding plate is placed at P, the grating G and the eyepiece of camera at O. Fig. 4 shows the plan of arrangement of the echelon used for producing these bands.[21] A is the light source, S the horizontal slit, C the collimator, E the echelon followed by a large flint glass prism, P the plate which is thrust half way into the beam and the telescope T, with the eyepiece or camera K. There were thirty elements, each about 1 cm. thick, in the echelon. Fig. 5 gives the plan used

[1] These numbers refer to corresponding numbers in the bibliography near the end of the paper.

with one of Michelson's gratings mounted Littrow. S is the slit, L the lens of about 20 ft. focal length (6 m.), G the plane grating, P the plane parallel plate, and O the observer. Fig. 6 is a print of an exposure made with the Hilger wave-length spectroscope. The source of light was the carbon arc. This is the entire visible spectrum with the red end



toward the left. The retarding plate was about 1 mm. in thickness and was placed at i, Fig. 2.

Fig. 7 is a spectrum in the second order with a small Rowland concave grating with a radius of curvature of about 180 cm. It is in a region near the D line of the solar spectrum. The plate was plane parallel about 7 mm. thick and was placed as in Fig. 3. There are 7 Talbot bands between the Fraunhofer lines D_1 and D_2 which appear about one fourth of the way across from the left end. Fig. 8 shows about 31 Talbot's bands between the two orange lines of sodium. The retarding plate was 31 mm. thick and the spectroscope was a 6 m. radius Rowland concave grating. On the original plate there are six exposures. In the exposure of which this is a part there are over 6,000 Talbot's bands. Fig. 9 is in the same region by the same means as Fig. 3 with less of the crater of the carbon arc focused on the slit. Both show the reversal of the sodium lines and there are three or four bands on each. Fig. 10 was taken with the echelon grating of 30 elements. The Talbot's bands were produced by a 30-mm. plane parallel plate. This print exhibits both the yellow and the green mercury lines with the bands making an angle with the spectrum lines, for the plate was not perpendicular to the beam of light.

Figs. 11, 12, 13, 14 and 15 show prints from exposures made with the echelon 46 mm. plate. nos. 11, 13, 14 and 15, the fringes make various

angles with the slit, while no. 13 the bands are parallel with the slit due to the 46-mm. plate being exactly perpendicular to the beam. No. 4 is of the indigo mercury line. The negative shows spectra of eleven orders with Talbot's bands in each. Usually the retarding plate is not of the correct thickness to give perfect interference fringes. If the plate is less than the best thickness whatever be the optical arrangement for analyzing the bands their appearance may be improved by cutting off a portion of the beam so as to make the two portions symmetrical and approximately equal in intensity. The Rowland concave grating with which Figs. 3 and 4 were taken has a resolving power of about 85,000 lines in first order. When a thickness of 77 mm. was used for producing the bands about 78 bands were visible between the centers of the two sodium lines. If a less thickness is used the fringes would be visible and could be improved by cutting down the two beams. To test this a large slit of heavy cardboard was constructed with wooden cross pieces which allowed a motion like parallel rulers. Its greatest width of opening was 15 cm. Placed between the grating and the ocular the best width for 50 mm. of glass was 10 cm., for 30 mm. about 4.4 cm., and for 10 cm. plate 1.7 cm. of opening. Thus the definition of the fringes was improved though the intensity of the illumination of the field was decreased.

With the concave grating the plates must be placed accurately perpendicular to the line joining the slit and the center of the grating. Two degrees out of the normal will make blurred fringes and a further slight change will cause them to disappear. On consideration of the way in which the interference is produced it is easy to see that inclining the plate displaces the retarded beam so that the rays which are recombined on the photographic plate are no longer congruent; hence there could be no regular interference.

With the echelon, on the contrary, as the plate is inclined from the perpendicular, the fringes are rotated with reference to the slit and appear as sharp or perhaps even more distinct at large angles than when parallel to the slit. The rotation may be increased until as many as thirty bands appear on the green mercury lines and a like number on each of the yellow and blue lines. Photographs were taken for both positions of single order and double order. The fringes seemed to be sharper for the position of double order or position of equal intensity. When making large angles with the slit the fringes have a close resemblance to the photographs taken by Nagaoka and Takamine by crossing Lummer-Gehrcke plate with an echelon. In some cases I have used as much as 92 mm. of glass for retarding one of the beams and have observed very sharp and clear bands. So that it is no longer appropriate to speak

Fig. 6.

Fig. 7.

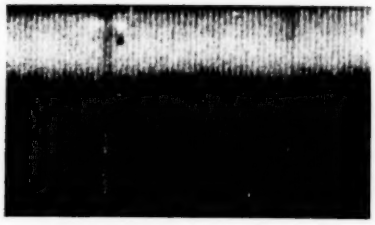


Fig. 8.

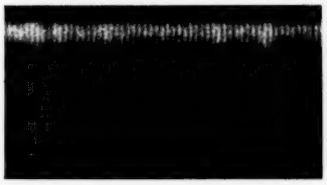


Fig. 9.

Fig. 10.

Fig. 11.

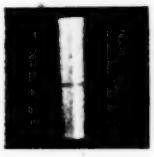


Fig. 12.

Fig. 13.

Fig. 14.

Fig. 15.

THOMAS E. DOUBT.

of the glass used for this purpose as a thin plate. In all cases where the thickness of the glass was 3 mm. or more I have used plane parallel glass plates. The fact that one millimeter of glass produces one Talbot band for about six Ångström units in the region of the D line suggests that we have an objective standard of resolving power for spectroscopes. The glass used was soft flint index about 1.6002.

There are many peculiar phenomena that can be noticed when one examines sunlight or an arc light by means of a powerful grating with the aid of Talbot's bands. This is especially true in the region about the sodium lines. Their reversal and distortion may be readily observed followed and measured by the position of the fringes which serve as a natural fixed micrometer, provided, of course, that the plate is not changed in temperature or position.

Theory of Talbot's Bands.

The discoverer of these bands, H. Fox Talbot, explained their formation on the undulatory theory as an interference of the unretarded beam with the beam retarded by the interposed plate. This explanation when followed out will give the correct number of bands for any part of the spectrum for a given thickness of plate.[1] Let t be the thickness, μ_1 its refractive index for wave-length λ_1 and, μ_2 for λ_2, $\lambda_1 > \lambda_2$ so that $\mu_2 > \mu_1$. The wave-lengths in the plate will be λ_1/μ_1 and λ_2/μ_2 and the retardations will be

$$\left(\frac{\mu_1 - 1}{\lambda_1}\right)t, \quad \text{and} \quad \left(\frac{\mu_2 - 1}{\lambda_2}\right)t$$

and the number of dark bands between λ_1 and λ_2 will be n where

$$n = t\left(\frac{\mu_2 - 1}{\lambda_2} - \frac{\mu_1 - 1}{\lambda_1}\right).$$

In photographing the Fraunhofer lines D_1 and D_2 with the Rowland concave grating forming the Talbot's bands with a thick plate μ_1 is nearly equal to μ_2 and if the thickness is known the mean refractive index can be determined with considerable accuracy with a single exposure. Dr. Wolcott Gibbs has shown that if this number is divided by the density a value is obtained which is constant for any given substance and is independent of the temperature.[10] Hence his interferential constant is

$$I = \frac{n}{\rho} = \frac{t}{\rho}\left(\frac{\mu_2 - 1}{\lambda_2} - \frac{\mu_1 - 1}{\lambda_1}\right),$$

where ρ is the density.

The simplest explanation of the lack of symmetry of Talbot's bands

has been offered by Schuster on the basis of the pulse theory of radiation. Since white light may be used to form the bands a single luminous impulse should be sufficient to produce them.[19] The action of a spectroscope upon such an impulse assumes its most elementary form when the analyzer is a plane reflection grating.[34] Let a plane light pulse fall normally upon a plane reflection grating. This grating may be considered to be made up of total reflection strips separated by strips which reflect no light. The light falls upon all reflection strips simultaneously. By Huygens' principle each point in the plane of the grating may be regarded as a secondary source. When the plane pulse falls on the grating the secondary pulses spread out from each reflecting strip and may be brought to a focus by a lens. The disturbance at the focus consists of a series of pulses following each other at intervals. If the axis of the lens makes an angle with the normal to the grating the pulses from one portion of the grating will arrive earlier at the focus than those from the other end of the grating. If interference is to be produced by the introduction of a transparent plate to retard half of the light from the grating it is evident that it must be introduced on that side so as to retard the pulses which arrive earlier. If introduced on the other side it retards those which already arrive too late to interfere.

The plate of the best thickness would be secured if the whole series of impulses is divided into two portions and the proper thickness of plate introduced so as to make the pulses arrive in pairs simultaneously. If n is the number of lines in the grating and λ the wave-length for the particular maximum corresponding to $\lambda = \epsilon \sin \theta$ where θ is the angle which the axis of the lens makes with the normal, ϵ is the grating space then the best thickness would be $\frac{1}{2}N\lambda$. The resolving power of the spectroscope is defined as

$$\frac{n_1}{n_1 - n_2} = R = \frac{\lambda}{d\lambda} = mN$$

for a grating where n_1 and n_2 are the frequencies of the two spectrum lines which are just separated, m the order of the spectrum, and N the total number of lines in the grating. Thus we see that the best thickness of the plate for a grating spectroscope is equal to $\frac{1}{2}N\lambda$ or one half of the wave-length by resolving power.

It has been shown by Lord Rayleigh that the resolving power of a prism is represented by the product of the effective thickness of the prism by the dispersive power or $t(d\mu/d\lambda)$ where t is the thickness and μ is the refractive index corresponding to the wave-length λ. Hence the best thickness for a prism spectroscope would be $\frac{1}{2}\lambda t(d\mu/d\lambda)$. Usually the retarding plate is not of the correct thickness to give perfect inter-

ference fringes. If the plate is less than the best thickness the fringes may be improved by cutting off a portion of the beam.

Another factor of great practical importance in resolving power is the width of the slit, for this determines the purity of the source. In his study of purity Schuster arrived at the conclusion that for a pure source we must use a width of slit equal to $f\lambda/4D$, where D is the diameter of the collimating lens and f its focal length. This width he calls the "normal slit." Further, if this width is doubled the loss of resolving power is only 6 per cent. but if the normal slit is eight times as wide the purity is only 45 per cent. of the normal slit.

The theory with a half-covered circular aperture has been given by Struve and we shall follow the earlier part of his work.[17] Let the observing telescope be adjusted for observing the Fraunhofer diffraction phenomena for a point source and a circular aperture. One half of the aperture is covered with a plane parallel plate which is transparent and correctly placed. The plane of the opening is that of xy, the origin of coördinates the middle point of the circle, the x-axis along the refracting edge of the plate and y-axis positive in the direction away from the covered semi-circle.

In the focal plane of the objective let ξ_1, η_1 be the coördinates of the geometric image of the origin and ξ, η the coördinates of the chosen point P and the focal distance is set equal to 1. Finally, let R be the radius of the circular aperture and δ be the phase difference between the rays which travel through the covered and uncovered portions of the aperture.

Then we may write the following expression for the intensity of the point P:

$$I = (C_1 + C_2)^2 + (S_1 + S_2)^2,$$

where

$$C_1 = \int\int \cos\left[\frac{2\pi}{\lambda}(\xi - \xi_1)x + \frac{2\pi}{\lambda}(\eta - \eta_1)y\right] dx\, dy,$$

$$S_1 = \int\int \sin\left[\frac{2\pi}{\lambda}(\xi - \xi_1)x + \frac{2\pi}{\lambda}(\eta - \eta_1)y\right] dx\, dy,$$

$$C_2 = \int\int \cos\left[\frac{2\pi}{\lambda}(\xi - \xi_1)x + \frac{2\pi}{\lambda}(\eta - \eta_1)y\right] dx\, dy,$$

$$S_2 = \int\int \sin\left[\frac{2\pi}{\lambda}(\xi - \xi_1)x + \frac{2\pi}{\lambda}(\eta - \eta_1)y\right] dx\, dy,$$

where C_1 and S_1 are integrated over all elements of the uncovered half of the aperture and C_2 and S_2 are for the covered half of the aperture.

Making the transformation to polar coördinates with

$$x = \frac{R}{z} r \cos \omega,$$

$$\frac{2\pi}{\lambda} R(\xi - \xi_1) = z \cos \omega,$$

$$y = \frac{R}{z} r \sin \omega,$$

$$\frac{2\pi}{\lambda} R(\eta - \eta_1) = z \sin \omega$$

Introducing these values and reducing

$$\int_0^{\pi} \cos [r \cos (\omega - \omega_1)] d\omega = \int_{-\omega_1}^{\pi-\omega_1} \cos (r \cos \omega) d\omega = 2 \int_0^{\pi/2} \cos (r \cos \omega) d\omega,$$

$$\int_0^{\pi} \sin [r \cos (\omega - \omega_1)] d\omega = \int_{-\omega_1}^{\pi-\omega_1} \sin (r \cos \omega) d\omega = 2 \int_0^{\omega_1} \sin (r \cos \omega) d\omega,$$

we obtain

$$C_1 = 2 \frac{R^2}{z^2} \int_0^z r \, dr \int_0^{\pi/2} \cos (r \cos \omega) d\omega,$$

$$S_1 = 2 \frac{R^2}{z^2} \int_0^z r \, dr \int_0^{\omega_1} \sin (r \cos \omega) d\omega,$$

and for

$$C_2 = C_1 \cos \delta - S_1 \sin \delta,$$

$$S_2 = - C_1 \sin \delta - S_1 \cos \delta,$$

and

$$I = 4 \left(C_1 \cos \frac{\delta}{2} - S_1 \sin \frac{\delta}{2} \right)^2.$$

It is remarkable what this expression exhibits. Convenient as a complete square it is valuable not only for a circular aperture but also for other apertures so long as the limiting line of the plate divides the opening into two symmetrical portions. In case the semicircles are not always symmetrical but are separated by a small strip of width e, then a correction must be made to the above, and in the place of δ write $\delta + e(z/R) \sin \omega_1$ wherever this variable appears in the formula.

As a special case if the geometrical image of the point P lies in the axis η_1, then $\xi = \xi_1 = 0$, and $\omega_1 = \pi/2$, the intensity at the point P_1 will be

$$J_0(z) = \frac{2}{\pi} \int_0^{\pi/2} \cos (z \cos \omega) d\omega,$$

and

$$zJ_1(z) = \int_0^z zJ_0(z)dz,$$

$$H_0(z) = \frac{2}{\pi}\int_0^{\pi/2} \sin\,(z\cos\omega)d\omega,$$

$$zH_1(z) = \int_0^z zH_0(z)dz,$$

$$I = \frac{4\pi R^2}{z^2}\left(J_1(z)\cos\frac{\delta}{2} - H_1(z)\sin\frac{\delta}{2}\right)^2,$$

where

$$z = \frac{2\pi}{\lambda R}(\eta - \eta_1).$$

$J_1(z)$ is the Bessel function of order unity which is well known in both of the following forms

$$J_1(z) = \frac{2}{\pi}\int_0^{\pi/2} \sin\,(z\sin\omega)\sin\omega d\omega = \frac{2}{\pi z}\int_0^{\pi/2} \cos\,(z\sin\omega)\cos^2\omega d\omega.$$

In a similar manner the unsymmetrical H functions may be derived. Through the double differentiation of $H_0(z)$ it yields

$$\frac{\partial^2 H_0(z)}{\partial z^2} + \tfrac{1}{2}\frac{\partial H_0(z)}{\partial z} + H_0(z) = \frac{2}{\pi z},$$

but

$$zH_0(z) = \frac{2}{\pi} - \frac{\partial\left(z\dfrac{\partial H_0(z)}{\partial z}\right)}{\partial z}$$

and from this according to definition

$$H_1(z) = \frac{2}{\pi}\left[1 - \int_0^{\pi/2} \cos\,(z\cos\omega)\cos\omega d\omega\right]$$

$$= \frac{4}{\pi}\int_0^{\pi/2} \sin^2\left(\frac{z\cos\omega}{2}\right)\cos\omega\, d\omega = \frac{4}{\pi}\int_0^{\pi/2} \sin^2\left(\frac{z\sin\omega}{2}\right)\sin\omega d\omega.$$

A second definite integral may be derived from this by use of Fourier series:

$$f(z) = \frac{2}{\pi}\int_0^{\infty} \sin\,(\alpha z)d\alpha \int_0^{\infty} f(\lambda)\sin\,(\alpha_1\lambda)d\lambda$$

where

$$f(z) = \frac{H_1(z)}{z}$$

$$\frac{H_1(z)}{z} = \frac{8}{\pi^2}\int_0^{\infty} \sin z\alpha d\alpha \int_0^{\pi/2} \sin\omega d\omega \int_0^{\infty} \frac{\sin\alpha\lambda\sin^2\left(\lambda\dfrac{\sin\omega}{2}\right)d\lambda}{\lambda}$$

but since

$$\int_0^\infty \frac{\sin \alpha\lambda \sin^2\left(\frac{\lambda \sin \omega}{2}\right)}{\lambda} d\lambda = \begin{cases} \frac{\pi}{4} \text{when } \sin \omega > \alpha, \\ 0 \text{ when } \sin \omega < \alpha, \end{cases}$$

$$\frac{H_1(z)}{z} = \frac{2}{\pi}\int_0^1 \sin \alpha z d\alpha \int_{\sin^{-1}\alpha}^{\pi/2} \sin \omega d\omega = \frac{2}{\pi}\int_0^1 \sin \alpha z \sqrt{1-\alpha^2}\, d\alpha,$$

and from this finally

$$H_1(z) = \frac{2z}{\pi}\int_0^{\pi/2} \sin (z \sin \omega) \cos^2 \omega d\omega.$$

Incidentally it may be mentioned that this furnishes the means for comparing the first form of $J_1(z)$ with the second and this method also allows us to exhibit the fundamental property of Bessel's functions as a very simple derivation.

$$\frac{2n}{z} J_n(z) = J_{n-1}(z) + J_{n+1}(z),$$

$$2\frac{\partial J_n(z)}{\partial z} = J_{n-1}(z) - J_{n+1}(z).$$

Next substitute for a single point source monochromatic light a series of independent sources whose geometrical image lies on the η axis. The points shall only be distinguished by their wave-lengths which from one side to the other may be taken to determine the specific intensity. On such a basis there would exist at each point an ideal vanishingly small spectrum at right angles to the division of the two portions of the objective. The intensity $I(\eta)$ of the points in the geometric image of this spectrum may be found through the summation of the intensity I for all possible values of η_1 which lie between the limits $-\infty$ and $+\infty$ so we may take for this

$$I(\eta) = \int_{-\infty}^{+\infty} J d\eta_1,$$

or through substitution of

$$z = \frac{2\pi R}{\lambda}(\eta - \eta_1),$$

$$I(\eta) = 2\pi\lambda R^3 \int_{-\infty}^{+\infty} \left(\frac{J_1(z)}{z}\cos\frac{\delta}{2} - \frac{H_1(z)}{z}\sin\frac{\delta}{2}\right)^2 dz.$$

Wherein the phase difference δ is a function of the wave-length and hence also a function η_1 or z which one may evaluate from Taylor's theorem in powers of z. If one designates with δ_0 the phase difference in the

point η we arrive at an approximation through

$$\delta = \delta_0 + (\eta_1 - \eta)\left(\frac{\partial\delta}{\partial\eta_1}\right)\eta = \delta_0 - \frac{\lambda}{2\pi R}\left(\frac{\partial\delta}{\partial\eta}\right)\frac{z}{\eta}$$
$$= \delta_0 + \alpha z.$$

In which the higher powers of z may be neglected. $J_1(z)/z$ and $H_1(z)/z$ are by nature very small quantities. Since this contribution to the value of δ would be very small so on the same ground we will assume that the specific intensity of the spectrum will be constant for each λ (and in the same expression for α) in the point η for which the wave-length is taken.

Omitting the factors $2\pi\lambda R^3$ the intensity of the point η for which the phase difference is δ_0

$$I(\eta) = \int_{-\infty}^{+\infty}\left[\frac{J_1(z)}{z}\cos\frac{az + \delta_0}{2} - \frac{H_1(z)}{z}\sin\frac{\alpha z + d_0}{2}\right]^2 dz.$$

From this the law of the intensity of the maxima and minima remain to be determined. Differentiating this expression for δ_0 and putting the differential coefficient equal to zero, one perceives that α varies slowly with η for δ_0 a constant; hence sin δ_0 = 0, from which it follows that the maximum and minimum respectively exist only where the path difference for the light rays are an odd or an even number of half waves.

The results obtained from the consideration of these integrals show that for a circular aperture it is impossible to obtain absolute maxima and minima otherwise the conclusions of Sturve are the same as those Lord Rayliegh has given in his article on Wave Theory in the Encyclopaedia Britannica. The explanation of the rotation of the fringes in using the echelon may be carried out according to Wood's suggestion that the plate used for forming Talbot's bands may be considered as an echelon of two elements.[21] The expression for the intensity as found by F. B. Galitzin is

$$I = C(n\alpha s)^2\left(\frac{\sin u}{u}\right)^2\left(\frac{\sin (nE/2)}{n\sin (E/2)}\right),$$

or

$$I = I_0 A(u)\varphi(v).$$

Where n is the number of steps in the echelon, s width, $u = \pi qs/\lambda$, $q = \sin(\varphi - \theta) + \sin\theta$, where θ is the angle of incidence and φ the angle of refraction

$$E = p + 2\pi\frac{q}{\lambda}s + 2\pi\frac{q}{\lambda} + \tan\varphi,$$

$$p = 2\pi\left(\frac{t}{\lambda}\right)\cos(\psi - \theta) - \mu\sec\varphi + \tan\varphi\sin(\varphi - \theta).$$

The expression for I above indicates that it is always positive, periodic and that it varies slowly with the angles. As one changes the angle of incidence θ the spectra shift in such a manner that there is one position for which the corresponding angle ψ is a minimum.

SUMMARY.

The resolving power of a spectroscope may be represented by the thickness of the plate of glass used for forming Talbot's bands. The best thickness represents the highest power for the given instrument. Examples are given here for resolving powers represented by less than one millimeter up to 92 millimeters in thickness. This furnishes further experimental evidence for the inference that for infinite resolving power infinite thickness would be required.

Stokes speaks of retardation of several hundred wave-lengths and says that: "This exalts our ideas of the regularity which must be attributed to the undulations."[5] My photographs show retardations of many thousand wave-lengths. Since the work of Lord Rayleigh, Guoy, Michelson, Fabry and Perot, and others little need be said about "regularity" in radiation. However, the phenomenon of Talbot's bands raises the question whether white light is not as much a unity as monochromatic light and withal just as simple? The retarding plate enables us to apply one test to all wave-lengths, while the spectroscope enables us to apply another test.

Much of the experimental work was done at the University of Chicago. I am pleased to express here my grateful acknowledgments to Professors Michelson, Millikan, and Gale, and the staff of Ryerson Physical Laboratory for the facilities so freely placed at my disposal.

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THE EMISSION OF ELECTRONS BY A METAL WHEN BOMBARDED BY POSITIVE IONS IN A VACUUM.

BY W. L. CHENEY.

IN the following paper, experiments are described which were undertaken with the object of ascertaining how the number of negative electrons emitted by a metal when bombarded by positive ions in a vacuum depends on the number, the velocity, and the nature of the positive ions.

It is well known that when a metal is bombarded by positive ions of sufficiently high velocity it emits negative electrons.[1] One of the first to show this was Villard,[2] who found that cathode rays are formed by positive ions impinging upon the cathode. He placed near the cathode a diaphragm having two small holes. As the tube was gradually exhausted, so long as the dark space did not extend to the diaphragm, the current flowed uniformly from the whole surface of the cathode. But after the dark space extended beyond the diaphragm the emission from the diaphragm became concentrated at two points opposite the holes in the diaphragm. In a high vacuum, two narrow rays passed from the cathode through the holes and produced a shadow of the diaphragm on the walls of the tube near the anode, showing that electrons were formed only when the positive ions hit the cathode.

J. J. Thomson[3] was the first to observe that when alpha rays from polonium bombard a metal, many slow speed electrons are emitted. Thomson named these negative electrons "delta rays," and concluded that their velocity was about that acquired in falling though a few volts only.

Füchtbauer has[4] shown that negative rays are given off when a metal is hit by canal rays, and that the velocity of the negative rays is independent of the velocities of the canal rays. He has further shown that some metals also reflect canal rays. When the velocity of the canal rays are due to a P.D. of 30,000 volts he found that all metals give off electrons for each canal ray particle in the same order as Volta's series; platinum giving least, and aluminum four electrons for each canal-ray particle.

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Campbell[1] found the speed of the delta rays to be independent of the speed of the alpha rays by which they were excited, and independent of the material from which the rays are emitted.

Bumstead[2] has found evidence that in addition to delta rays, positive ions are also produced when alpha rays impinge upon a metal in a very high vacuum. These, however, appear to come from the layer of absorbed gas on the metal.

More recently, McLennan and Found[3] have investigated the problem by measuring the number of delta rays emitted from zinc when bombarded with alpha rays in a high vacuum. They found an emission of three electrons per alpha particle from freshly scraped zinc. This effect diminished with the lapse of time and ceased altogether for a while when the zinc was freshly coated in vacuo with a deposit from zinc vapor.

THE EXPERIMENTS.

The method employed in this investigation was to obtain positive ions by heating different salts, such as potassium sulphate, in a vacuum, on a strip of platinum, through which an alternating current was passed. A metal plate was placed near the strip and the positive ions made to bombard it by giving it a negative charge. The current between the strip and plate could be easily measured by a sensitive galvanometer.

The ratio of the negative electrons emitted by the plate to the positive ions striking it, could be found in the following manner. Let C_1 represent the thermionic current carried by positive ions and negative electrons, so that

$$C_1 = C_+ + C_-. \tag{1}$$

Now, if a transverse magnetic field be set up which will stop the electrons by causing them to curve back upon the metal plate but will not stop the positive ions,

$$C_2 = C_+. \tag{2}$$

Dividing (1) by (2)

$$\frac{C_1}{C_2} = 1 - \frac{C_-}{C_+}$$

or

$$\frac{C_-}{C_+} = \frac{C_1}{C_2} - 1. \tag{3}$$

Let N_+ be the number of positive ions striking the plate per second, and N_- be the number of negative electrons given off from the plate per

[1] Campbell, Phil. Mag., Vol. 22, p. 276 (1911), and Vol. 23, p. 46 (1912).

[2] Bumstead, Am. Journ. of Sci., Vol. 36, pp. 91–108 (1913).

[3] McLennan and Found, Phil. Mag., Vol. 30, p. 491 (1915).

second. Then $C_+ = N_+e_+$ and $C_- = N_-e_-$ where e_+ and e_- are the charges on the positive and negative ions respectively. But since $e_+ = e_-$, we have the desired relation,

$$\frac{N_-}{N_+} = \frac{C_-}{C_+}. \tag{4}$$

The procedure was to observe, for a given P.D., first C_1 (directly with the galvanometer), then C_2, then C_2 with the magnetic field reversed, and finally C_1, again. This was done to obviate any fluctuations arising from a change of heating current. In most cases, however, this was really unnecessary, for the initial and final values of C_1 did not differ appreciably. From the means of C_1 and C_2, C_-/C_+ was calculated. Representative values to illustrate this are incorporated in Table I.

TABLE I.

Pt Cathode, K + ions.

P. D. Volts.	Galv. Deflections in Mm. (1.65×10^{-10} Amp.).						C_1/C_2.	C_-/C_+.
	C_1.	C_2.	C_2 (H Reversed.)	C_1.	C_1 Mean.	C_2 Mean.		
146	131	130	130	130	130.5	130	1.004	.004
190	147	146	146	147	147	146	1.006	.006
250	154	152	156	158	156	154	1.012	.012
280	160	157	157	159	159.5	157	1.015	.015
350	166	163	163	166	166	163	1.018	.018
400	166	162	162	165	165.5	162	1.021	.021
475	172	168	168	172	172	168	1.023	.023
525	168	164	164	169	168.5	164	1.025	.025
560	152	148	148	152	152	148	1.025	.025

The magnitude of H (the magnetic field), necessary to stop the negative ions without stopping the positive, could be calculated from a formula given by J. J. Thomson[1] for determining e/m when using a magnetic field to stop ions passing between parallel plates; viz.,

$$\frac{e}{m} = \frac{2V}{H^2d^2},$$

where e/m is the ratio of the charge in the ion to its mass, V the potential difference between the plates, H the magnetic field, and d the distance between the plates.

Calculations according to this formula are exhibited in the following table.

[1] J. J. Thomson, Conduction of Electricity through Gases (1906), p. 219.

TABLE II.

Nature of Ions.	Electrons.	K + Ions.	Li + Ions.	Pb + Ions.
e/m.	1.7×10^7 E.M.U.	$10^4/39$ E.M.U.	$10^4/7$ E.M.U.	$10^4/85$ E.M.U.
H(calculated) necessary to just stop ions at 100 volts (10^{10} E.M.U.)	50	9,000	4,000	13,000
H(calculated) necessary to just stop ions at 600 volts (6×10^{10} E.M.U.)	110	21,000	9,000	32,000
H employed in the experiment	..	400	400	400

The effect produced by H was further tested by choosing a definite P.D. and varying H over a considerable range (100–1,000). As no change occurred in the diminution of the leak it was concluded that even with as high a value of H as 1,000 units positive ions were not being deflected unless it were at very low potentials, such as 10–50 volts. No observations of C_-/C_+ were made at these low P.D.s, the reason being that any diminution of current caused by H was too small to be detected or did not exist at all. In nearly all the observations here recorded H was 400 units.

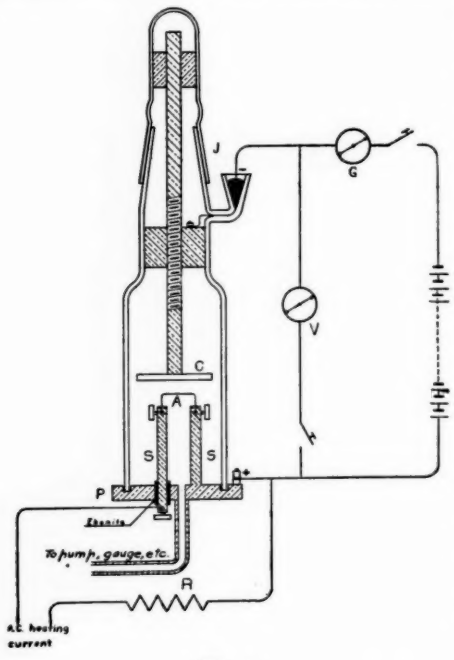

Fig. 1.

The apparatus is shown diagrammatically in Fig. 1. It consisted of a glass tube about 4 cm. in diameter cemented to a brass plate P with sealing wax, and supported between the poles of a large electromagnet. SS were two brass rods, supporting the narrow strip of platinum, A. One of the rods passed through an ebonite plug and was thus insulated from P. The strip of platinum could be heated to any desired temperature by passing a 60-cycle alternating current through it. It could be made the anode by connecting to the positive terminal of a battery of "Tungsten Ever-Ready" cells (capable of giving nearly 600 volts), while C, the metal plate, was made the cathode. C was carried by a micrometer screw and could be moved up and down by turning the ground joint J, so that the distance AC could be varied as desired. Some ob-

servations made by moving C up and down and noting the current showed that the leak across the gap AC decreased slightly as the distance AC was increased. This is shown in the following table.

TABLE III.

Dist. Between Electrodes (Mm.).	Current × (1.65 × 10⁻¹⁰ Amperes) (P.D. = 146 Volts).						
1.....	160	160	150	134	127	180	210
2.....	160	158	—	132	125	178	208
4.....	154	156	—	—	—	—	202
5.....	150	155	140	127	120	170	200
6.....	150	154	138	127	118	168	—
8.....	145	148	—	—	—	166	198
10.....	142	145	134	126	112	165	196

In the subsequent observations the distance between the platinum strip and the metal plate was kept at about one centimeter.

The potential difference between A and C was measured by a Kelvin Electrostatic Voltmeter, and the thermionic current was measured by a Leeds and Northrup sensitive galvanometer (sensibility = 1.65×10^{-10} ampere per mm. deflection).

The greatest difficulty throughout the experiment was the securing of a good vacuum. A Gaede rotary mercury pump was used in series with a box pump. When a sensitive McCleod Gauge indicated no gas pressure, the vacuum was put to further test by means of an induction coil whose terminals were placed across the gap AC and the pump kept running

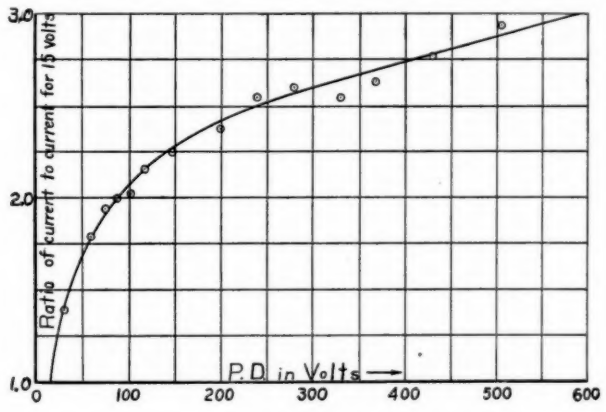

Fig. 2.

until no fluorescence appeared in the tube and the spark preferred to pass through the air outside. The mercury or oil vapors which might

have been in the apparatus were frozen out by means of CO_2 snow. Finally, the pump was kept running throughout a series of observations. The apparatus was kept dry by means of P_2O_5, and the vacuum was washed out from time to time with a little dry air.

To test the variation of the thermionic current with the change of potential difference, a double throw switch was placed in the circuit so that the leak for any P.D. could be compared quickly with the leak for 15 volts. This is illustrated in Fig. 2, in which the P.D.s are plotted as abscissæ and the ratios of the leak for given P.D.'s to the leak for 15 volts as ordinates. It is worthy of note that with low potential differences the leak rose rapidly with the increase of P.D. until about 150 volts where it approached saturation.

To find C_-/C_+, observations were made with aluminium and platinum as cathodes and K_2SO_4, Li_2SO_4, and Rb_2SO_4 respectively on the hot platinum strip A, as a source of positive ions. For each particular salt and metal, a great many observations were made. Table IV. shows representative values for a number of observations in a particular case,

TABLE IV.

Pt Cathode, K + Ions.

P.D. (Volts).	C_-/C_+.												Mean C_-/C_+.
146	.005	.005	.007	.004	.002	.005	.005	.003	.005	.003	.003	.007	.0045
190	.005	.012	.012	.006	.008	.005	.007	.009	.006	.006	.009	.004	.007
250	.012	.012	.012	.012	.013	.012	.020	.012	.011	.011	.011	.011	.0125
280	.012	.015	.012	.015	.018	.018	.016	.015	.013	.012	.013	.013	.014
350	.018	.032	.018	.013	.012	.020	.012	.012	.016	.019	.013	.016	.017
400	.023	.014	.023	.021	.025	.018	.025	.015	.013	.021	.016	.020	.0195
475	.023	.020	.022	.018	.030	.024	.015	.022	.021	.017	.020	.016	.021
525	.020	.030	.025	.025	.026	.022	.022	.025	.028	.018	.028	.018	.024
560	.020	.026	.032	.025	.028	.028	.028	.031	.018	.018	.028	.029	.025

while Table V. gives the mean values for all the salts and metals used. By an inspection of Table V. and Fig. 3, it is seen that the largest effect occurs in the case of lithium and the smallest with rubidium, while that from potassium lies between the others; the values for all three being greater with aluminium than with platinum.

Some preliminary experiments showed that the effect decreased somewhat after the apparatus had stood evacuated for several days. Consequently, all the results recorded in Tables I., IV. and V. are those taken after the apparatus had been allowed to stand evacuated for several days. Some observations were made with Li_2SO_4 on the hot strip, and aluminium as cathode, after the apparatus had been standing for nearly a

TABLE V.

Mean Values of G/C_+.

P.D. Volts.	Al Cathode.			Pt Cathode.		
	K_+.	Li_+.	Rb_+.	K_+.	Li_+.	Rb_+.
74	.004	....	....	....	.006	....
115	....	....	....	....	....	.0016
125	.008	....	....	....	....	....
146	.011	....	....	.0045	.013	.002
177	....	.010	....	....	....	....
190	....	....	....	.007	.017	.003
215	.018	.020	....	....	....	....
235	....	....	....	....	....	.005
250	....	....	....	.0125	....	....
265	....	....	....	....	.019	....
275	.025	.058	.008	.014	.007	....
300	....	....	....	.024	....	....
340	.037	.093	.014	....	....	....
350	....	....	....	.017	....	.008
360	....	....	....	....	.026	....
375	.047	....	....	....	....	....
390	....	....	....	....	.030	....
400	....	....	....	.0195	....	.009
450	.067	.138	.022	....	....	....
475	.080	....	....	.0207	.036	....
535	....	.214	.036	....	....	....
560	.092	....	....	.025	.043	.012

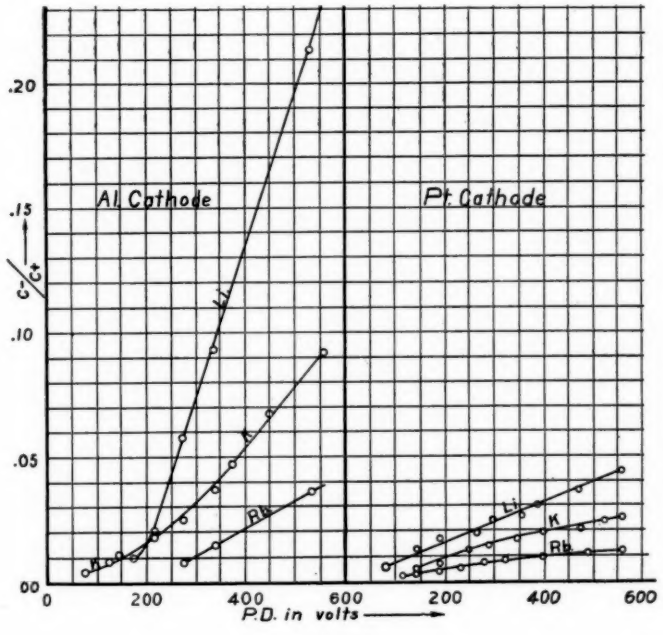

Fig. 3.

month. Comparing the results with those obtained after a week's time a slight decrease is noticeable (Fig. 4).

It seems quite likely, therefore, that part of the effect, at least, is due to gas absorbed by the metal. However, some tests were made with

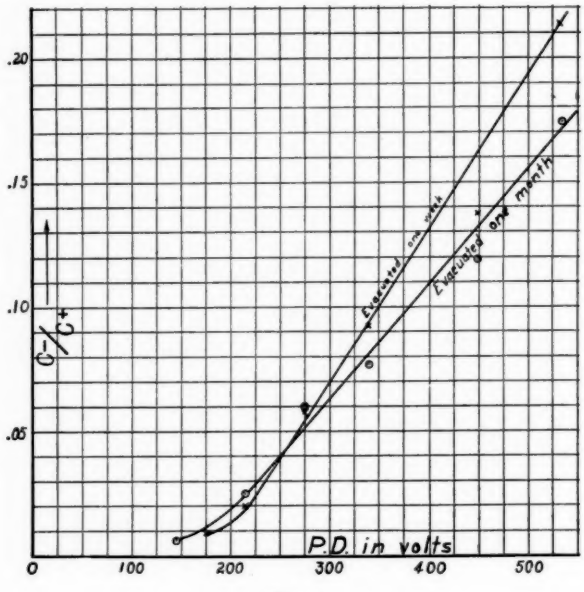

Fig. 4.

platinum cathode and rubidium, after heating the metal plate for several hours in air at a low pressure by bringing it in contact with the hot platinum strip. The results obtained (Table VI.), although somewhat erratic,

TABLE VI.

Showing the Effect of Gas. (Pt Cathode, Rb^+ Ions.)

P.D. (Volts).	C_-/C_+.		
	As in Table V.	After Heating in Air.	After Heating in Hydrogen.
115	.0016	....	....
146	.002	.000	.0025
190	.003	.004	.005
235	.005	.0065	.0085
265	....	.007	....
275	.007	....	.009
350	.008	.008	.010
390	....	.009	....
400	.009	.009	.012
485	.011	.0095	.014
560	.012	.010	.017

approximate to those obtained prior to the test with air. A similar test was made using hydrogen instead of air. This time the values of C_-/C_+ were slightly greater than previously, indicating that the platinum had possibly soaked up some of the hydrogen. These values, however, are not as greatly in excess as one might expect, so it appears that the gas was only slightly absorbed.

APPLICATION TO DISCHARGE IN GASES.

Consider the dark space in a discharge of electricity through a gas at low pressure and suppose no positive ions striking the cathode during a particular time interval. Some of these positive ions striking the cathode will set free electrons but only those which have fallen through a long enough free path to acquire sufficient velocity. Let the *mean* free path be represented by λ. Let n be the number during this time interval which have free paths greater than a length x. Then $dn = -\beta n dx$, where β represents the number of collisions a positive ion makes in going one centimeter.[1] Therefore, on integrating, $n = n_0\epsilon^{-\beta x} = n_0\epsilon^{-x/\lambda}$. The number having free paths between x and $x + dx$ is therefore given by

$$dn = \frac{n_0}{\lambda}\epsilon^{-x/\lambda}dx.$$

Let γ be the ratio of the number of electrons emitted from the cathode to the number of positive ions producing them. γ is a function of the velocity of the positive ions as has been found in the experiments described above. $\bar{\gamma}$, the average value of γ, is given by the expression

$$\bar{\gamma} = \frac{1}{n_0}\int d n\gamma = \frac{1}{n_0}\int_{x_1}^{x_2}\left(\frac{n_0}{\lambda}\epsilon^{-x/\lambda}\right)\gamma dx. \tag{5}$$

By considering Fig. 3, it can be seen that the curve representing the values of $\gamma(= C_-/C_+)$ is practically a straight line and can be expressed analytically by

$$\gamma = aV - b, \tag{6}$$

where V is the potential difference, a the slope of the curve and b a constant. Equation (6), however, holds only for positive values of γ.

If we assume, on the basis of Aston's experiments,[2] that there is no appreciable difference of potential between the cathode and the adjacent gas, we may write

$$V = \int_0^x X dx,$$

[1] Townsend, loc. cit.

[2] F. W. Aston, Proc. Roy. Soc., *A*, 84, p. 526, 1911.

where X is the electric force. In the dark space X varies uniformly, being a maximum at the cathode and a minimum at the negative glow, so that $X = A - Bx$, where A and B are constants. At the negative glow $X = 0$, and $A = BD$, where D is the length of the dark space. Hence

$$V = B\int_0^x (D - x)dx = B\left(Dx - \frac{x^2}{2}\right). \tag{7}$$

When $\gamma = 0$, $V = b/a$. Call this particular value of V, V'. Then

$$V' = B\int_0^{x'} (D - x)dx = BDx' - \frac{Bx'^2}{2}, \tag{8}$$

where x' is the distance in the dark space representing the paths through which the positive ions fall under the P.D. of V' volts before impinging on the cathode.

To determine B, take

$$K = \int_0^D xdx,$$

where K is the "normal" cathode fall of potential. From this we get

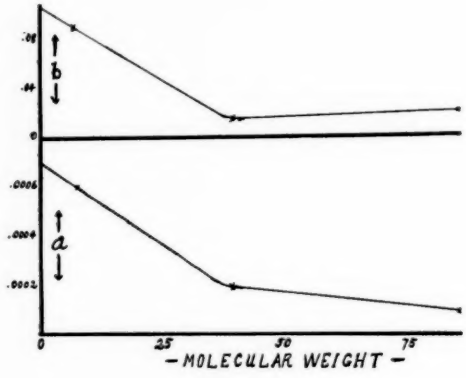

Fig. 5.

$B = 2K/D^2$ and equations (7) and (8) now become

$$V = \frac{2K}{D^2}\left(Dx - \frac{x^2}{2}\right), \tag{7'}$$

$$V' = \frac{2K}{D^2}\left(Dx' - \frac{x'^2}{2}\right). \tag{8'}$$

Equation (6) becomes

$$\gamma = a\frac{2K}{D^2}\left(Dx - \frac{x^2}{2}\right) - b. \tag{6'}$$

and finally (5) becomes

$$(5') \qquad \bar{\gamma} = \frac{1}{\lambda}\int_{x'}^{D} \epsilon^{-x/\lambda}\left[a\frac{2K}{D^2}\left(Dx - \frac{x^2}{2}\right) - b\right]dx.$$

Suppose now we consider a special case, viz., an aluminium cathode and a discharge through hydrogen with a gas pressure corresponding to one mm. of Hg. Investigators have found that under these conditions, $D = 1$ cm. approximately, $K = 200$ volts (nearly), and λ (according to Meyer) is .013 cm. for the hydrogen molecule. The values of V', a, and b, corresponding to the different kinds of positive ions, are found from Fig. 3, when aluminium was used as cathode.

Positive Ions.	Mol. Wt.	a.	b.	V'.
Li.....	7	.0006	.09	150
K.....	39	.0002	.015	75
Rb.....	85	.0001	.020	200

Similar values for hydrogen are found by plotting the above values of a and b against the molecular weights (Fig. 5) and extrapolating. a is found to be .007, $b = .104$, and $V' = 150$ volts (nearly). Substituting the values of K, V', and D, in (8') and solving, $x' = .5$ cm. Substituting for the various constants their numerical values, (5') may be simplified to

$$(5'') \qquad \bar{\gamma} = \frac{1}{\lambda}\int_{.5}^{1} \epsilon^{-x/\lambda}\left[.28\left(x - \frac{x^2}{2}\right) - .104\right]dx.$$

Integrating

$$(9) \qquad \bar{\gamma} = \epsilon^{-x/\lambda}[-.28(x+\lambda) + .14\{x^2 + 2\lambda(x+\lambda)\} + .104]_{.5}^{1}.$$

Upon evaluating, $\bar{\gamma}$ is found to be of the order 10^{-17}, which, of course, is negligible.

While Aston has found no appreciable difference of potential between the cathode and the gas, others[1] have found a considerable drop in the potential right at the cathode. Under these conditions,

$$V_\lambda = \int_0^x X dx + V_0,$$

where V_0 denotes the fall of potential right at the cathode and

$$K = \int_0^D X dx + K_0,$$

where K_0 denotes the value of V_0 in the case of the " normal " cathode fall.

[1] C. A. Skinner, PHYS. REV., June and Aug., 1915, W. L. Cheney, PHYS. REV., Feb., 1916; W. E. Neuswanger, PHYS. REV., Feb., 1916.

Thus equations (7′), (8′), (6′), and (5′) become modified to

$$V = \frac{2(K - K_0)}{D^2}\left(Dx - \frac{x^2}{2}\right) + V_0, \tag{10}$$

$$V' = \frac{2(K - K_0)}{D^2}\left(Dx' - \frac{x'^2}{2}\right) + V_0, \tag{11}$$

$$\gamma = a\left[\frac{2(K - K_0)}{D^2}\left(Dx - \frac{x^2}{2}\right) + V_0\right] - b, \tag{12}$$

$$\bar{\gamma} = \frac{1}{\lambda}\int_{x'}^{D} \epsilon^{-x/\lambda}\left\{a\left[\frac{2(K - K_0)}{D^2}\left(Dx - \frac{x^2}{2}\right) + K_0\right] - b\right\} dx. \tag{13}$$

x' is found to be .3 cm., K_0 (from the experiments of the writer) is nearly 90 volts; the values of the other constants are the same as above. $\bar{\gamma}$ is in this case of the order 10^{-10}.

Thus, it appears from calculations based on either Aston's or Skinner's experiments, that in the case of the " normal " cathode fall of potential in a discharge of electricity through hydrogen at low pressure extremely few electrons are set free from the cathode. The above calculation, however, is subject to error since one is not quite sure of the value V'. As already stated above, it was difficult to obtain any accurate observations of γ for low values of V and one is not certain that the curves in Fig. 3, which we have assumed to be nearly straight lines, do not become asymptotic to the V-axis. At any rate, the calculation shows that $\bar{\gamma}$ is very small.

Skinner,[1] working with the " normal " cathode fall in hydrogen at low pressures, calculated $\bar{\gamma}$ to be of the order 10^{-4}, for an aluminium cathode. The writer,[2] making use of Skinner's theory, found under similar conditions the same order of magnitude for $\bar{\gamma}$.

Skinner's theory, however, does not take into account the collisions of positive ions with the molecules of the gas. H. A. Wilson,[3] taking into the account the ionization by collision of the positive ions has shown that $\bar{\gamma}$ is probably small.

Townsend consider $\bar{\gamma} = 0$ except at very low pressures when high potentials are necessary. Aston[4] has made some investigations under these conditions. Here is one set of values which he obtained when using aluminium cathode in hydrogen: $V = 700$ volts, $D = 2.09$ cm., $p = .157$ mm. of Hg. From these values x' works out to be .25 cm. and $\lambda = .083$ cm. Applying these to equation (5′) $\bar{\gamma}$ is found to be of the order 10^{-2}.

[1] C. A. Skinner, loc. cit.

[2] W. L. Cheney, loc. cit.

[3] H. A. Wilson, PHYS. REV., Sept., 1916.

[4] F. W. Aston, Proc. Roy. Soc., *A*, Vol. 87, p. 437.

It appears, therefore, that $\bar{\gamma}$ increases very rapidly as the pressure is diminished. The experiments described in this paper show that in the case of a thermionic current in a vacuum γ is appreciable for lower values of the P.D. corresponding to the "normal" cathode fall in hydrogen.

Summary.

1. The magnitude of the thermionic current in a vacuum corresponding to various P.D.s has been compared with the thermionic current corresponding to a P.D. of 15 volts.

2. The ratio of the number of electrons leaving the cathode to the number of positive ions striking it has been found with positive ions of different velocities and for two different metals, viz., aluminium and platinum. It has been found that this ratio depends on the velocity of the positive ions.

3. It has been found that the effect is diminished somewhat after the metal has stood in a vacuum for some time and increased slightly after it had stood in hydrogen. It appears, then, that the effect is at least partially due to gas in the metal.

4. The ratio of the number of electrons emitted from the cathode to the number of positive ions bombarding it has been calculated for the case of a discharge in hydrogen.

The writer wishes to express his indebtedness to Professor H. A. Wilson, at whose suggestion this investigation has been undertaken, and whose interest and kindly counsel have been very valuable in surmounting many difficulties.

The Rice Institute,
Houston, Texas,
March, 1917.

THE FLUORESCENCE OF FOUR DOUBLE NITRATES.

BY H. L. HOWES AND D. T. WILBER.

IN an early paper by Professors Nichols and Merritt[1] on the luminescence of the uranyl salts it is noted that the effect of the water of crystallization on the spectrum of the uranyl nitrate is to shift the bands slightly in the direction of the longer wave-lengths. In a more recent paper[2] the effect on the fluorescence of the nitrate has been studied in detail and it is shown that the spectra of the anhydrous salts and of specimens with $2H_2O$, $3H_2O$ and $6H_2O$ differ from each other profoundly as regards the position and the grouping of the bands.

On the other hand there is good reason to think that crystal form has an important bearing upon the structure and arrangement of fluorescent spectra. In a study of the polarized spectra of four of the double uranyl chlorides,[3] which crystallize in the triclinic system it was found that the spectra of these salts were almost identical in arrangement and in the absolute position, relative intensity and resolution of their bands. In an independent investigation of frozen solutions of various uranyl salts two or more strikingly different spectra from uranyl nitrate were obtained by varying the rate of freezing the aqueous solution.[4] A year ago the present authors made a brief study of seven forms of the sodium uranyl phosphate and concluded that only when crystals were found could a resolved spectrum be obtained by cooling.[5]

The object of this brief paper is to throw a little more light on the rôle played by crystal structure. We had hoped to be able to produce two crystals having different crystal systems but identical chemical formulæ. In this we have failed, but have two crystals with nearly the same formula and different crystal systems.

The two pairs of double nitrates studied are mono-ammonium uranyl nitrate $NH_4UO_2(NO_3)_3$; di-ammonium uranyl nitrate $(NH_4)_2UO_2(NO_3)_4\ 2H_2O$; the mono-potassium uranyl nitrate $KUO_2(NO_3)_3$ and the di-potassium uranyl nitrate $K_2UO_2(NO_3)_4$.

[1] Nichols and Merritt, PHYS. REV. (1), XXXIII., p. 375, 1911.
[2] Nichols and Merritt, PHYS. REV. (2), XIV., p. 125, 1917.
[3] Nichols and Howes, PHYS. REV. (2), VIII., p. 364 (1916).
[4] H. L. Howes, PHYS. REV. (2), VI., p. 206, 1915.
[5] H. L. Howes and D. T. Wilber (2), VII., p. 394, 1916.
[6] Nichols and Merritt, PHYS. REV. (2), IX, p. 125, Feb., 1917.

The mono-ammonium salt, which crystallizes from a solution of the two component salts in concentrated nitric acid was described by Meyer & Wendel[1] and crystallographically by Steinmetz.[2] The crystals are of the trigonal system with an axial ratio of $a : c = 1 : 1.0027$.

The di-ammonium salt crystallizes from a slightly acid water solution of the two salts in which the ammonium nitrate is in excess of that required for the mono-ammonium salt. This salt was at first thought to be the α modification of ammonium uranyl nitrate made by Rimbach[3] and measured by Sachs[4] but an examination of the spectrum of the α modification so called proved that it was simply uranyl nitrate hexahydrate. The crystals analyzed by Rimbach were probably the mono-ammonium form as this sometimes forms in the same solution. The crystals of the di-ammonium salt belong to the monoclinic system. The mono-potassium salt crystallizes from nitric acid solution in the rhombic system as described by Steinmetz with axial ratio $a : b : c = .8541 : 1 : .6792$.

The di-potassium salt crystallizes with reluctance; but when seeded

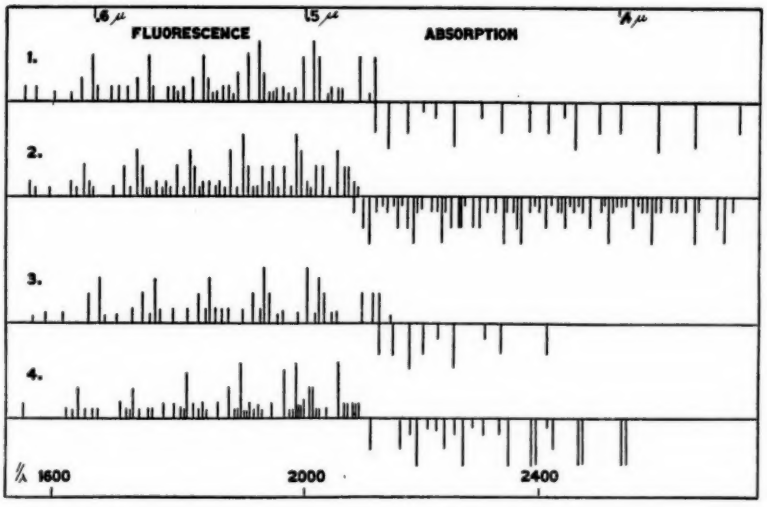

Fig. 1.

1. Fluorescence and absorption spectra of Mono-ammonium uranyl nitrate, $NH_4UO_2(NO_3)_3$.
2. Di-ammonium uranyl nitrate, $(NH_4)_2UO_2(NO_3)_4.2H_2O$.
3. Mono-potassium uranyl nitrate, $KUO_2(NO_3)_3$.
4. Di-potassium uranyl nitrate, $K_2UO_2(NO_3)_4$.

[1] Meyer and Wendel, Ber. d. d. Ch. Ges., Vol. 36, 4055, 1903.
[2] Steinmetz, Groth's Chem. Krys., II., p. 150.
[3] Rimbach, Ber. d. d. Ch. Ges., Vol. 37, 472, 1904.
[4] Sachs, Zeitschr. f. Krys., Vol. 38, 497, 1904.

from an acid aqueous solution, it forms in beautiful, fluorescent crystals of the monoclinic system. The axial angle $\beta = 90° \pm$ and the axial ratio $a : b : c = .6394 : 1 : .6190$. The composition is different from that of the di-ammonium salt, since it lacks the water of crystallization.

TABLE I.

Series in the Fluorescence Spectrum of Mono-ammonium Uranyl Nitrate.

	1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).
A	1,797.6	88.3	*G*	1,573.9	89.1	*K*	1,859.1	89.7
	1,885.9	86.2		1,663.0	88.0		1,948.8	87.0
	1,972.1	86.2		1,751.0	87.2		2,035.8	
	2,058.3			1,838.2	87.7			
				1,925.9	87.9	*L*	1,602.1	90.6
B	1,629.4	88.7		2,013.8	87.5		1,692.7	88.2
	1,718.1	87.8		2,101.3			1,780.9	88.1
	1,805.9	87.5					1,869.0	84.5?
	1,893.4	90.7	*I*	1,670.0	88.0		1,953.5	87.5
	1,984.1			1,758.0	87.7		2,041.0	
				1,845.7	88.5			
D	1,555.5	89.6		1,934.2	88.1	*M*	1,704.2	86.2
	1,645.1	88.9		2,022.3	87.9		1,790.4	88.4
	1,734.0	87.0		2,110.2			1,878.8	86.1
	1,821.0	88.1					1,964.9	85.7
	1,909.1	87.8	*J*	1,852.5	88.9		2,050.6	
	1,996.9	89.2		1,941.4				
	2,086.1							

Series in the Absorption Spectrum of Mono-ammonium Uranyl Nitrate.

	1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).
a	2,132.8	74.3	*d*	2,163.1	74.2	*i*	2,111.0	76.7
	2,207.1	73.4		2,237.3	75.9		2,187.7	74.9
	2,280.5	75.8		2,313.2	73.7		2,412.5	74.8
	2,356.3	73.8		2,386.9			2,562.1	
	2,430.1	72.4						
	2,502.5		*g*	2,469.1	76.4×2			
				2,621.9	72.0			
				2,693.9				

Both visual and photographic measurements of the spectra were taken, and since they agreed well, were averaged together. When possible the absorption spectrum was obtained by transmitted light. The crystals from an acid solution were of a deeper green color than those from a water solution, which necessitated grinding to about .4 mm. thickness to make them sufficiently transparent. Since the immersion in liquid air spoiled a crystal many crystals of each form had to be prepared.

Since, as is usual with the uranyl salts, we have in these spectra series

TABLE II.

Series in the Fluorescence Spectrum of Di-ammonium Uranyl Nitrate.

	1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).
A	1,773.6	83.8	*F*	1,637.7	84.8	*J*	1,664.5	84.2
	1,857.4	84.4		1,722.5	84.3		1,748.7	83.5
	1,941.8	84.1		1,806.8	84.3		1,832.2	86.0
	2,026.4			1,891.1	85.4		1,918.2	86.3
				1,976.5	85.4		2,002.5	82.1
B	1,695.6	83.9		2,061.9			2,084.6	
	1,779.5	84.9	*G*	1,564.0	86.0			
	1,864.4	84.9		1,650.0	83.3	*K*	1,754.1	83.9
	1,949.3			1,733.3	83.5		1,838.0	85.1
C	1,786.4	85.2		1,816.8	84.0		1,923.1	84.9
	1,871.6	83.6		1,900.8	84.5		2,008.0	
	1,955.2	84.4		1,985.3	83.2	*L*	1,595.4	85.6
	2,039.6			2,068.5			1,681.0	83.4
D	1,628.8		*I*	1,572.9	84.9		1,764.4	83.5
	1,713.1	84.3		1,657.8	83.8		1,847.9	83.3
	1,796.3	83.2		1,741.6	82.9		1,931.2	84.7
	1,880.8	84.5		1,824.5	84.0		2,015.9	
	1,965.2	84.4		1,908.5	84.7			
	2,050.2	85.0		1,993.2	83.7			
				2,076.9				

Series in the Absorption Spectrum of Di-ammonium Uranyl Nitrate.

	1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).
a	2,114.8	70.6	*e*	2,131.0	70.0	*k*	2,092.9	71.0
	2,185.4	69.3		2,201.0	67.8		2,163.9	69.3
	2,254.7	69.1×4		2,268.8	69.6		2,233.2	70.8
	2,531.0	69.3×2		2,338.4	69.9		2,304.0	69.8
	2,669.5			2,408.3	69.1		2,373.8	69.8
b	2,178.1	70.6		2,477.4	68.2		2,443.6	67.5
	2,248.7	72.6		2,545.6	69.1×2		2,511.1	73.6
	2,321.3	71.3		2,683.8			2,584.7	72.8
	2,392.6		*g*	2,140.8	70.1		2,657.5	
c	2,124.5			2,210.9	68.2	*l*	2,102.3	71.6
	2,332.5	69.3×3		2,279.1			2,173.9	71.2
	2,401.3	68.8	*i*	2,077.3	70.9		2,245.1	71.7
	2,472.2	70.9		2,148.2	69.8		2,316.8	
d	2,344.9	70.0		2,218.0	73.0	*α*	2,384.8	69.4
	2,414.9	69.8		2,291.0	67.5		2,454.2	69.1
	2,484.7	68.1		2,358.5	70.7		2,523.3	69.4
	2,552.8	68.4		2,429.2	68.5		2,592.7	
	2,621.2			2,497.7	69.3	*δ*	2,422.3	68.6
				2,567.0			2,490.9	68.8
			j	2,154.2	70.1		2,559.7	68.4
				2,224.3	70.6×2		2,628.1	
				2,365.4	70.7	*γ*	2,538.1	68.1
				2,436.1	68.1		2,602.2	
				2,504.2				

of constant frequency intervals, the tables contain the frequencies of the bands, where the frequency unit is of such that for $\lambda = 5{,}000$ Å.U. the table reading is $1/\lambda = 2{,}000$ units. The average positions given should never be in error more than two frequency units.

The relation between fluorescence and absorption series is of the same nature as that previously found to exist in the spectra of the uranyl salts. Fig. 1 indicates the four spectra.

By referring to Table I. fluorescence series "*I*" will be seen to consist of six bands beginning with the red band at 1,670.0 fre-

TABLE III.

Series in the Fluorescence Spectrum of Mono-potassium Uranyl Nitrate.

	1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).
B	1,725.3	88.5	*G*	1,589.8	85.0	*K*	1,615.8	84.6
	1,813.8	86.6		1,674.8	87.2		1,700.4?	89.8
	1,900.4	87.7		1,762.0	86.5		1,790.2	87.3
	1,988.1			1,848.5	86.4		1,877.5	87.0
D				1,934.9	86.5		1,964.5	85.8
	1,569.1	86.8		2,021.4	86.4		2,050.3	86.0
	1,655.9	86.8		2,107.8			2,136.3	
	1,742.7	87.5						
	1,830.2	86.1				*I*	1,683.2	86.2
	1,916.3	87.5	*J*	1,867.6	87.5		1,769.4	87.5
	2,003.8	86.7		1,955.1	88.1		1,856.9	86.8
	2,090.5			2,043.2			1,943.7	86.8
							2,030.5	
F	1,754.1	88.2					2,118.2	
	1,842.3	86.2						
	1,928.5	87.2						
	2,015.7							

Series in the Absorption Spectrum of Mono-potassium Uranyl Nitrate.

	1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).
d	2,167.0	71.3	*i*	2,117.7	71.9	*k*	2,140.3	72.9
	2,238.3	75.5		2,189.6			2,213.2	74.7
	2,313.8	72.3					2,287.9?	
	2,386.1							

quency units and continuing to the blue band at 2,110.2. Absorption series "*i*" in Table I. has the first member at 2,111.0 which almost agrees in position with the last fluorescence band, of series "*I*", but the intervals between the absorption bands are seen to be shorter than those between fluorescence bands. The nomenclature of the fluorescence and absorption series is so chosen that the series which are related have the same letter, *e. g.*, "*A*" is related in the

same manner as above to "*a*," "*B*" to "*b*," etc., although the "reversing" band is not generally present. Consider series "*A*" for example, which ends with band 2,058.3; the first absorption band of series "*a*" is not at 2,058.3 but at 2,132.8. If we note that the absorption interval is

TABLE IV.

Series of the Fluorescence Spectrum of Di-potassium Uranyl Nitrate.

	1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).
B	1,775.2	86.7	*F*	1,723.8?	84.8	*I*	1,651.5	86.1
	1,861.9	87.0		1,808.6	85.5		1,737.6	86.2
	1,948.9	85.7		1,894.1	86.2		1,823.8	87.4
	2,034.6			1,980.3	88.5		1,911.2	87.3
				2,068.8			1,998.5	87.0
D	1,621.3	87.7					2,085.5	
	1,708.0	85.7	*G*	1,554.0	86.4			
	1,793.7	86.3		1,640.4	87.2	*J*	1,831.8	87.7
	1,880.0	86.9		1,727.6	85.6		1,919.5	87.5
	1,966.9	86.8		1,813.2	85.8		2,007.0	
	2,053.7			1,899.0	87.2			
				1,986.2		*K*	1,663.3	88.5
E	1,631.6	86.3					1,751.8	86.0
	1,717.9	84.8	*H*	1,903.7	86.5		1,837.8	87.3
	1,802.7	86.6		1,906.8			1,925.1	86.8
	1,889.3	86.5		1,989.7			2,011.9	
	1,975.8	86.4		1,993.8				
	2,062.2			2,075.8	86.6	*L*	1,670.8	87.0
				2,080.7			1,757.8	86.8
							1,844.6	86.8
							1,931.4	86.9
							2,018.3	

Series in the Absorption Spectrum of Di-potassium Uranyl Nitrate.

	1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).		1/λ.	Δ(1/λ).
d	2,196.6	72.7	*h*	2,152.6	71.7	*l*	2,105.0	74.9
	2,269.3			2,224.3			2,179.9	
e	2,210.9	74.8	*k*	2,169.2	70.9	*l'*	2,253.4	71.7
	2,285.7			2,240.1	70.8		2,325.1	71.8
				2,310.9	71.7		2,396.9	
f	2,369.4	75.0		2,382.6				
	2,444.4					*δ*	2,361.6	76.2
							2,437.8	75.4
							2,513.2	

about 74 units and add 74 to 2,058.3 we obtain 2,132.3, which is the reason for classifying series "*a*" as we have.

In Table II. there are three extra absorption series which are not so intimately related to fluorescence series, hence they are designated by

the Greek letters. This is also true of series "l'" and "δ" in Table IV. In the tables of series of the mono-ammonium and mono-potassium salts there are not sufficient absorption series to match with fluorescence series, but there are no extra absorption series which do not join. In the

TABLE V.

Average Intervals.

Mono-ammonium Uranyl Nitrate.

Fluorescence series	*A*	*D*	*G*	*I*
	86.6	88.3	87.7	88.1
Absorption series	*a*	*d*	*g*	*i*
	73.7	74.5	74.2	75.1
Ratio of fluorescence to absorption	*A/a*	*D/d*	*G/g*	*I/i*
	1.18	1.19	1.18	1.17

Di-ammonium Uranyl Nitrate.

Fluorescence series	*A*	*B*	*C*	*D*	*E*	*G*	*I*	*J*	*K*	*L*
	84.4	84.4	84.3	84.5	85.0	83.7	83.9	83.8	84.8	84.0
Absorption series	*a*	*b*	*c*	*d*	*e*	*g*	*i*	*j*	*k*	*l*
	69.2	71.6	69.8	68.9	68.7	68.8	69.6	69.7	70.9	71.5
Ratio of fluorescence to absorption	*A/a*	*B/b*	*C/c*	*D/d*	*E/e*	*G/g*	*I/i*	*J/j*	*K/k*	*L/l*
	1.22	1.18	1.21	1.23	1.22	1.22	1.21	1.20	1.20	1.18

Mono-potassium Uranyl Nitrate.

Fluorescence series	*D*	*I*	*K*
	86.9	87.2	86.6
Absorption series	*d*	*i*	*k*
	73.2	71.9	74.1
Ratio of fluorescence to absorption	*D/d*	*I/i*	*K/k*
	1.18	1.16	1.17

Di-potassium Uranyl Nitrate.

Fluorescence series	*D*	*E*	*F*	*H*	*K*	*L*
	86.6	86.2	87.2	86.5	86.9	86.9
Absorption series	*d*	*e*	*f*	*h*	*k*	*l*
	72.7	74.8	75.0	71.7	71.3	74.9
Ratio of fluorescence to absorption	*D/d*	*E/e*	*F/f*	*H/h*	*K/k*	*L/l*
	1.19	1.16	1.16	1.21	1.22	1.16

di-ammonium spectrum there is a related absorption series for each fluorescence series.

The study of related series is made confusing by the presence of absorption series extending into the fluorescence region; and, vice versa, many fluorescence series, if extended, fit absorption bands. A special

study of the over-lapping region is needed to determine better the relation between fluorescence and absorption series. Such a study is now in progress. As to the completeness of classification of bands into series it can be said that not a fluorescence or absorption band of any of the salts fails to fit into one of the constant frequency series.

In the study of the spectrum of the uranyl nitrate by Merritt[1] each fluorescence series has constant intervals between the bands which are the same for all series; in the case of the absorption series, however, the interval is not the same for all series. In our study of the double nitrates we find an unmistakable variation in the fluorescence intervals as well

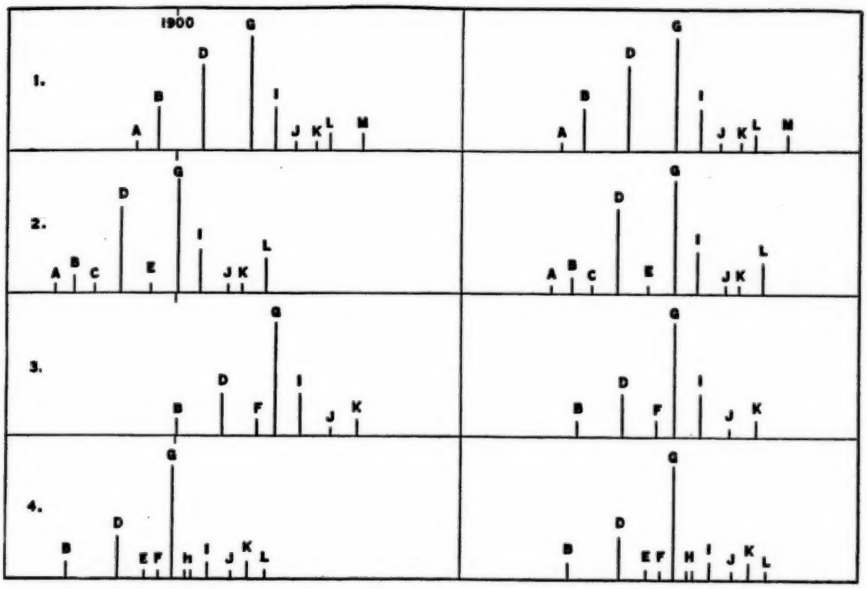

Fig. 2.

A single group from each of the four spectra.

1. Mono-potassium uranyl nitrate.—Trigonal.
2. Di-potassium uranyl nitrate.—Monoclinic.
3. Mono-ammonium uranyl nitrate.—Rhombic.
4. Di-ammonium uranyl nitrate.—Monoclinic.

The bands occupy their natural positions in the left-hand panel, but have their strongest bands in vertical alignment in the right-hand panel.

as in the absorption. In the mono-ammonium nitrate the interval varies from 86.4 for series "*M*" to 89.0 for series "*B*." In the di-ammonium nitrate the interval varies from 83.7 for "*G*" to 85.0 for "*E*." In the mono-potassium nitrate spectrum the interval varies between 86.4 for "*G*" and 87.9 for "*J*." In the di-potassium nitrate the interval varies between 86.2 for "*E*" and 87.6 for "*J*."

[1] Nichols and Merritt, PHYS. REV. (2), Vol. IX., p. 125, Feb., 1917.

The variation of the interval in the absorption series is of the same order of magnitude; *e. g.*, an extreme variation of 1.4 in the mono-ammonium, 3.5 in the di-ammonium, 4.3 in the mono-potassium and 4.4 in the di-potassium nitrate. In this connection it was thought to be of interest to compare the ratios of related fluorescence and absorption intervals. In Table V. these ratios are given. The ratios are nearly constant for the mono-ammonium and mono-potassium uranyl nitrates; but differ in the case of the other two salts.

That the crystal system to which a salt belongs is an important factor in determining the position of the bands can be seen in Fig. 2. In the left-hand panel a single group is shown in its natural position; in the right-hand panel the strongest bands of each group are placed in the same vertical line, to show the resemblance in grouping. This grouping is probably due to the fact that all four belong to the same chemical family. If we compare this grouping with that of the uranyl nitrate spectra studied by Merritt we find little resemblance, hence the grouping is probably characteristic of the double uranyl nitrate family. In the left-hand panel it will be seen that the second and fourth groups occupy almost identical positions, while the first and third occupy positions which differ from one another, and from the second or fourth. As has previously been stated the second and fourth groups belong to the monoclinic crystal systems, the first to the trigonal and the third to the rhombic system. Since all four spectra vary slightly in their frequency intervals the relative positions would change slightly if we compared homologous groups in the other end of the spectrum, but this gradual and slight shifting would not change the general condition which indicates that the absolute position of a group is largely determined by the crystal system. This is not entirely new, as the four triclinic crystals of the double uranyl chlorides studied by Nichols and Howes exhibited spectra which were as nearly coincident as could be expected of salts which vary in molecular weight.

Again, in the case of the uranyl nitrate, the crystals of the hexahydrate were of the rhombic system, while those of the trihydrate and dihydrate were of the triclinic system. In spite of slight shifts due to changing molecular weight the strong bands of the two spectra produced by the crystals of the triclinic system agree fairly well, while the strong bands of the spectrum produced by the rhombic crystal reside in entirely different positions.

There is one more bit of evidence which adds weight to the above view. The chemical formulæ of the two potassium salts are more nearly alike than those of the two ammonium salts since the di-ammonium salt has two molecules of water of crystallization, while the other salts have none,

yet there is a greater difference between the third and fourth spectra than there is between the first and second spectra.

Summary.

1. The spectra of the double uranyl nitrates resemble those of the previously studied uranyl salts in that the bands can be arranged in series having constant frequency intervals.

2. These intervals while constant for any given series are different for different series.

3. In the mono-ammonium uranyl nitrate and the mono-potassium uranyl nitrate the ratio of the interval of a fluorescence series to the interval of the absorption series which joins that fluorescence series is approximately a constant.

4. Although the grouping of the bands shows a strong family resemblance in the four spectra yet the absolute position of a group is largely determined by the crystal system.

Physical Laboratory of Cornell University,
April 6, 1917.

THE REVERSAL OF THE HALL EFFECT IN ALLOYS.

BY ALPHEUS W. SMITH.

IN studying the Hall effect in bismuth-tin alloys von Ettingshausen and Nernst[1] made the interesting observation that for a certain strength of the magnetic field there is a reversal in the direction of the Hall electromotive force. For small values of the magnetic field the direction of this electromotive force is the same as that in bismuth. As the magnetic field is increased the Hall electromotive force at first increases, passes through a maximum, and sinks to zero, after which it reverses its direction and increases continuously. To account for this reversal von Ettingshausen and Nernst put forth the following suggestion which they did not undertake to verify.

In Fig. 1 let the plate AB have a current of electricity flowing from A to B. If there is a magnetic field perpendicular to the plane of the plate, two phenomena are observed simultaneously. If the direction of the flow of the Amperian magnetizing current is that indicated by the arrow on the circle, the upper edge of the plate becomes negative with respect to the lower and its temperature is increased above that of the lower. There is thus set up a temperature gradient from the top to the bottom of the plate. If the plate is perfectly insulated the flow of heat produced by this temperature gradient is just balanced by the heat generated at the top in excess of that at the bottom of the plate. The difference of temperature thus established is a measure of the von Ettingshausen effect and is given by the equation

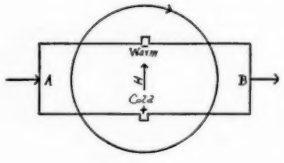

Fig. 1.

$$\Delta t = {}_eT_h \frac{Hi}{d},$$

where i is the current in absolute units, H the magnetic field and d the thickness of the plate in centimeters. The Hall electromotive force is given by the equation,

$$E = {}_eT_e \frac{Hi}{d}.$$

[1] Ann. der Phys., 33, p. 474, 1888.

If it happens that the plate is imperfectly insulated from its surroundings there will be a flow of heat from the medium into the plate. This flow will enter the bottom where the plate is colder than the surroundings and leave it at the top where it is warmer than the surroundings. It seems to me that a measure of the maximum flow to be expected from this source may be had by regarding the actual temperature gradient in the plate reversed in direction, so that there is in the plate from bottom to top a flow of heat in the direction opposite to that which would result from the observed temperature gradient in the plate. This flow is indicated by the vertical arrow in Fig. 1.

It has been observed in these same alloys that if the flow of electricity is replaced by a flow of heat as indicated in Fig. 2 and if the thermo-electromotive force between two points A and B on the longitudinal axis of the plate is compensated, under the magnetic action there is set up a difference of potential between these points. Of course this potential difference may be interpreted as a change in thermo-electromotive force on account of the magnetic action. This difference of potential is not reversed with the magnetic field. In bismuth and these alloys it is found that its direction is such that the hot end of the plate is positive with respect to the cold end. The magnitude of this potential difference is given by the equation,

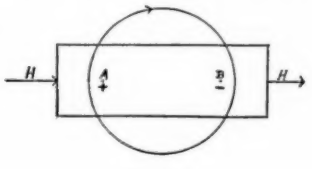

Fig. 2.

$$E = LH\frac{\partial t}{\partial x},$$

where $\partial t/\partial x$ is the temperature gradient in the plate.

Now suppose that on account of the transverse flow of heat which von Ettingshausen and Nernst assume to be associated with the Hall effect, there is established this longitudinal effect between the top and the bottom of the plate. This would give an electromotive force which would be superposed on the ordinary Hall electromotive force. These two electromotive forces would be opposite in direction.

The observed electromotive force would be their algebraic sum. Let E = the Hall electromotive force between the top and bottom of a plate one centimeter in thickness when a current of one electromagnetic unit is flowing in it. Let Δt = the corresponding difference of temperature between the top and bottom of a plate one centimeter wide. Let E_2 = the longitudinal potential difference established in the plate by the magnetic action when the temperature gradient is one degree per cm.

Let E_1 = observed Hall electromotive force. Then,

$$E_1 = E - E_2\Delta t.$$

Whether this will account for the reversal is obviously a question of the relative magnitudes of these potential differences.

More recently Becquerel[1] observed that it is possible to obtain this reversal of the Hall electromotive force in a plate cut from a crystal of bismuth. In such plates the magnetic field at which the reversal takes place depends on the direction of the crystalline axis with respect to the magnetic field and the plane of the plate. Chapman[2] has observed that the Corbino effect is reversed in these bismuth-tin alloys under essentially the same conditions under which the Hall electromotive force is reversed. In the Corbino effect the plates are discs. The current enters at the center of the disc, flows along the radius and leaves at the periphery. The magnetic action causes the current to have a component normal to the radius of the disc. The effective electromotive force producing this component of the current is essentially a Hall electromotive force. Under the conditions under which it is produced it is not clear that the temperature effect to which von Ettingshausen and Nernst attribute the reversal can arise.

The author has observed that the Hall effect is reversed in bismuth-tin alloys for alternating currents with a frequency of 60 cycles per sec. It would seem that for alternating currents of this frequency the accompanying thermal effects on which the explanation of von Ettingshausen and Nernst is based, would scarcely have sufficient time to become effective. The fact that the reversal occurs under these conditions shows that either the explanation is not to be ascribed to the accompanying thermal effects or that these thermal effects manifest themselves more quickly than is ordinarily supposed.

In view of these considerations it seemed worth while to study the reversal of the Hall effect in these alloys, extending the observations to higher field than those used by von Ettingshausen and Nernst. The reversal has also been observed in alloys of bismuth and lead. The principal point of interest is to see whether this reversal of the Hall effect as ordinarily defined can be caused by the superposition of some of the allied phenomena on the Hall electromotive force.

The continuous curves in Fig. 3 which have been plotted from the observations of von Ettinghausen and Nernst show the relation between the observed Hall electromotive force and the magnetic field for two

[1] Comp. Rendus, 154, pp. 1795–1798, 1912.

[2] Phil. Mag., 32, pp. 303–326, 1916.

alloys of bismuth and tin. The ordinates are the Hall electromotive forces in a plate one cm. thick with a current of one electromagnetic unit in it. One of these alloys contained 0.95 per cent. tin; the other contained 1.46 per cent. tin. The dotted curves in these figures show the potential difference to be expected from the thermal flow assumed by von Ettingshausen and Nernst. It must be remembered that the measure of this flow of heat is obtained by assuming the temperature difference between the top and bottom of the plate, arising from the magnetic action on the electric current, is reversed. This flow is then at right angles to the direction in which the electric current is flowing. On account of this flow of heat the magnetic field produces a difference of potential between the top and bottom of the plate which would be either added to, or subtracted from the Hall electromotive force. The ordinates for these curves are obtained by multiplying the longitudinal potential difference for a particular magnetic field when the temperature gradient is one degree per cm. in the plate by the difference of temperature between the top and the bottom of the plate due to the Ettingshausen effect when the plate is one centimeter wide and one centimeter thick and is traversed by a current of one electromagnetic unit. It is obvious from these curves that these longitudinal potential differences are much too small to account for the observed reversal of the Hall electromotive force.

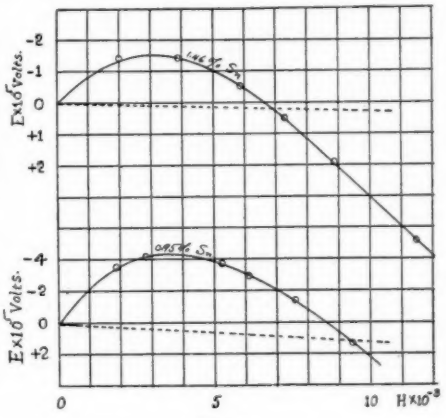

Fig. 3.

Since the properties of tin and lead are very similar it seemed probable that alloys of bismuth and lead would show the reversal of the Hall electromotive force which had been observed by von Ettingshausen and Nernst in alloys of bismuth and tin. In order to test this, three alloys of bismuth and lead were prepared by fusing together in known proportions by weight Baker's analyzed bismuth and Kahlbaum's pure lead. These alloys were then cast in the form of rectangular plates about 0.1 cm. thick, 1.5 cm. wide, and 4.0 cm. long, with narrow arms projecting from the middle of each of the longer sides of the rectangle. These arms were sufficiently long to project outside of the more intense part of the magnetic field. To these arms were soldered the lead wires

which were joined to the galvanometer on which the Hall electromotive force was observed. By filing these arms near the edge of the plate they could be shifted until they were nearly on the same equipotential line. Heavy strips of copper were soldered along the ends of the plate and served as electrodes by which the current entered or left the plate. Care was taken that the lines of flow be as nearly as possible parallel to the edges of the plate. With such a plate it is unnecessary to make correction for the thermo-electromotive force arising from the temperature difference between the top and bottom of the plate set up by the magnetic action, for the lead wires where they are joined to the plate are of the same material as the plate and the temperature difference which results from the magnetic action will not produce an appreciable thermo-electromotive force. Essentially no temperature change occurs where the arms are soldered to the lead wires. This form of plate avoids the necessity of making that correction for the Ettingshausen effect which is necessary where the lead wires are soldered directly to the plate.

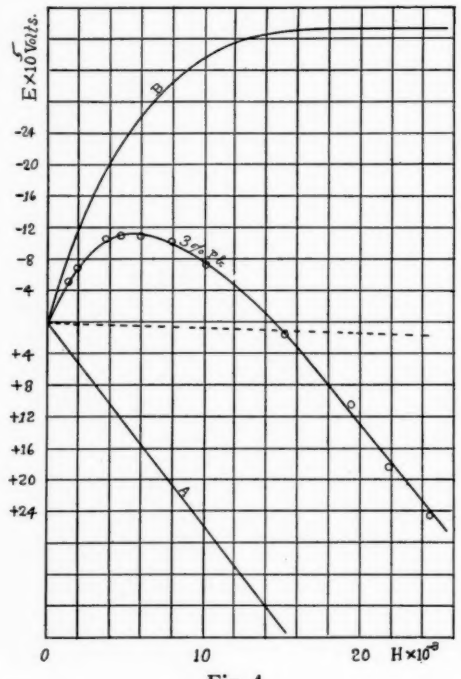

Fig. 4.

For further details with respect to the measurement of the Hall effect in these alloys reference is made to an earlier paper on "The Variation of The Hall Effect in Metals with Change of Temperature."[1] Except for minor details the arrangement of the apparatus and the method of taking observations were the same as used in that paper.

In order to know whether the idea suggested by von Ettingshausen and Nernst would account for the reversal of the Hall effect which was observed in these alloys of bismuth and lead, it was necessary to know the Ettingshausen effect and the longitudinal potential difference in them. Observations had already been made on the Ettingshausen effect in these alloys in connection with the study of this effect in several series of alloys. A description of the method used and of the results obtained

[1] PHYS. REV., 30, p. 1, 1910.

in that investigation were published in the PHYSICAL REVIEW, N. S., Vol. VIII., p. 82, 1916. The necessary data for the present purpose were taken from the results given in that paper.

For the investigation of the longitudinal thermomagnetic potential difference the same method was used which the author has previously described in the PHYSICAL REVIEW, N. S., Vol. II., p. 383, 1913. In the present investigation no essential departures from the details of that method were made.

The curves showing the results obtained in the study of these three alloys of bismuth and lead have been given in Figs. 4, 5 and 6. These

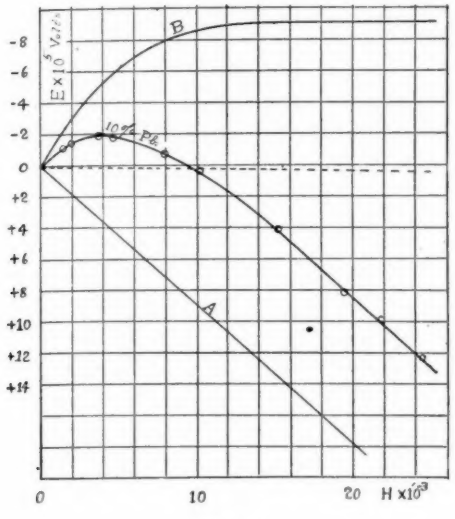

Fig. 5.

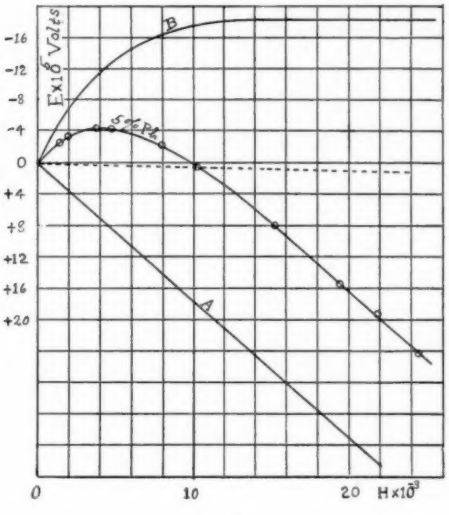

Fig. 6.

alloys contain 3 per cent., 5 per cent., and 10 per cent. of lead respectively. From the observations on the Ettingshausen effect and on the longitudinal potential difference, the potential differences to be expected from the transverse flow of heat assumed by Ettingshausen and Nernst, have been calculated. These potential differences have been plotted for ordinates in the dotted curves given in Figs. 4, 5 and 6. Here as in the alloys studied by von Ettingshausen and Nernst the potential differences which might arise from the longitudinal potential effect are much too small to account for the reversal of the Hall effect.

Becquerel has suggested that such curves, showing the relation between the Hall electromotive forces and the magnetic fields may be split up into two curves,—Curve *A*, a straight line passing through the origin and Curve *B* which arises to a fixed value after which it is parallel to the horizontal axis. The sum of the ordinates of these two curves gives the

ordinates of the observed curve. This analysis regards the Hall effect as made up of two parts which are opposite in sign. Of course this analysis is arbitrary, as a number of other pairs of curves may be chosen. It is, however, suggested by the fact that for the larger values of the magnetic field the Hall electromotive force is proportional to the magnetic field. Furthermore, this analysis is helpful in the present discussion because it shows the least correction that must be added to the Hall electromotive force to account for the observed reversal. The slope of Curve *A* is more than forty times that of the corresponding dotted curve showing the possible correction arising from the longitudinal potential difference. Even admitting the validity of the assumption made by von Ettinghausen and Nernst concerning the reversed flow of heat it is necessary to conclude that their suggestion cannot account for the reversal of the Hall effect.

In Fig. 7 the Hall electromotive forces in bismuth and in three alloys

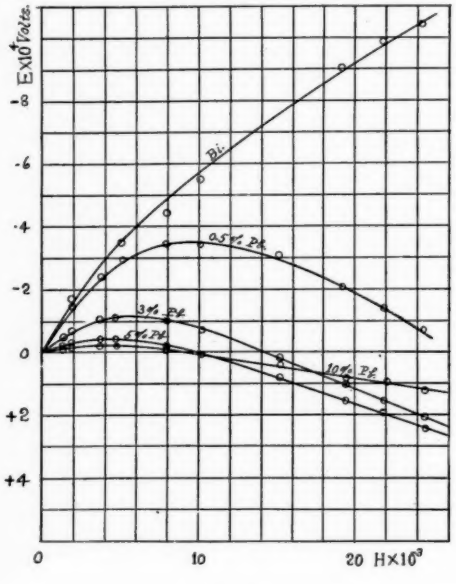

Fig. 7.

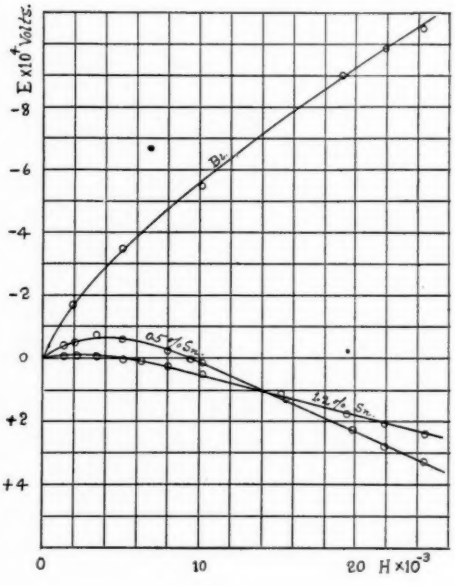

Fig. 8.

of bismuth and lead have been plotted against the magnetic fields producing them. In Fig. 8, similar curves are shown for bismuth and two alloys of bismuth and tin. The characteristics of each of these sets of curves are essentially the same. Small additions of either lead or tin to bismuth causes a rapid decrease in the Hall effect. For any one of the alloys the Hall electromotive force at first has the same direction as in

bismuth. With increasing field it increases to a maximum, reverses its direction and then increases proportional to the magnetic field.

It is necessary to conclude from these and other observations that there has yet been no satisfactory suggestion advanced to account for the reversal of the Hall effect under the conditions considered in this paper. The explanation of this interesting fact is probably to be looked for in the structure of the alloy, particularly in the interstices between the vibrating atoms.

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RESISTANCE AND MAGNETIZATION.

BY C. W. HEAPS.

IN its present condition the theory of free electrons in metals explains qualitatively the general aspects of most of the observed phenomena, but fails to account quantitatively for some of the more special experiments. In particular, the Hall effect and the other galvanomagnetic phenomena exhibit peculiarities in different metals which are difficult to bring into line with the theory. The difficulty appears to arise because of complex and little understood molecular actions inside the metal which the theory does not take into account. For this reason the above mentioned class of phenomena deserves special consideration. Furthermore, the explaining of the motion of free electrons in magnetized metals involves the explanation of magnetostriction and of magnetism. The results of some previous experiments[1] made by the writer have indicated very clearly that the resistance change induced in iron and nickel by magnetization is very intimately associated with the structural changes producing magnetostriction. In order to bring out this relationship more clearly it seemed important to make experiments on the influence of structure on the resistance change of metallic conductors when placed in magnetic fields. The present paper describes these experiments.

In interpreting the results it will be necessary to consider the theoretical side of the question. On the usual theory of free electrons in a metal it is found that when a magnetic field acts at right angles to an electric current in the metal the expression for the change of resistance involves the term $H^2(e/m)^2T^2$ multiplied by a numerical factor. Here H is the magnetic field and T the free period of an electron. The numerical factor by which the above expression is multiplied has been determined as $\frac{1}{4}$[2] and as $\frac{1}{12}$[3], and there has been some disagreement as to whether or not the theoretical resistance change is an increase or a decrease.

In order to clarify our ideas it will be of advantage to consider the separate factors which enter to change the magnitude of the electric current in a wire when this wire is magnetized. It is ordinarily assumed that free electrons in a metal move about with high velocity very much

[1] PHYS. REV., VI., p. 34, 1915.

[2] E. P. Adams, PHYS. REV., XXIV., p. 428, 1907.

[3] O. W. Richardson, "The Electron Theory of Matter."

like gas molecules but collide with molecules of metal instead of with each other. When an electric field acts on these electrons a drift motion is superposed on the ordinary velocity of agitation of the electrons and it is this drift motion which constitutes the electric current. It has been shown[1] that the velocity with which electrons drift under these conditions is given by the equation

$$U_0 = X\frac{e}{m}\frac{\lambda_0}{V}, \tag{1}$$

where U_0 is the drift velocity in the direction of the electric force X, λ_0 is the mean free path of the electrons, and V is the velocity of agitation of the electrons.

We have now to consider the effect of a magnetic field on this drift velocity in two special cases as follows: (1) when H is applied parallel to the drift velocity (the longitudinal effect), and (2) when H is applied perpendicular to the drift velocity (the transverse effect). In case (1) it is clear that the only way in which H can change U_0 is by affecting the mean free path of an electron. Such an effect of H on λ_0 is to be expected, for the existence of magnetostriction implies an alteration of intermolecular distances and a consequent change in λ_0. It has also been pointed out[2] that the paths made by electrons between collisions would on the average be curved in the case where a magnetic field acts, and hence would be changed in length for this reason also. If λ_h is the mean free path in the longitudinal magnetic field we may write $\lambda_h - \lambda_0 = \delta\lambda$

$$\frac{\delta U}{U_0} = \frac{\delta\lambda}{\lambda_0}, \tag{2}$$

where δU is the increase in U_0 produced by the increase $\delta\lambda$ in λ_0. The electric current, I, is proportional to U_0 in case the magnetic field does not affect the number of electrons per unit volume, so that if δI represents the increase of current produced by $\delta\lambda$ we have

$$\frac{\delta I}{I} = \frac{\delta\lambda}{\lambda_0}.$$

For the increase of resistance we may write

$$\frac{\delta R}{R} = -\frac{\delta\lambda}{\lambda_0}. \tag{3}$$

According to this equation a longitudinal magnetic field will produce an increase of resistance when the mean free path of the electrons is decreased by H.

[1] Langevin, Ann. de Chim. et de Phys., 28, p. 336, 1903.

[2] Richardson, "Electron Theory of Matter."

In case (2) we have an added effect to consider. Townsend has shown[1] that if λ_0 is unchanged by H the drift velocity in a transverse magnetic field is given by

$$U_h = \frac{U_0}{1 + \omega^2 T^2}, \tag{4}$$

where $\omega = H(e/m)$, and T, the mean value of the time between collisions, is equal to λ_0/V. We have seen, however, that a magnetic field may be expected to change λ_0. If we let $\lambda_h - \lambda_0 = d\lambda$ be this change produced in λ_0 by a transverse magnetic field we must use λ_h instead of λ_0 in equation (4). We thus have, approximately,

$$U_h(1 + \omega^2 T^2) = U_0 \frac{\lambda_h}{\lambda_0},$$

$$\frac{dU}{U_0} = \frac{U_h - U_0}{U_0} = \frac{d\lambda}{\lambda_0} - \omega^2 T^2. \tag{5}$$

The change of electric current is proportional to the change in U_0 so that this expression gives the increase of current produced by a transverse magnetic field. For the increase of resistance we may write

$$\frac{dR}{R} = \omega^2 T^2 - \frac{d\lambda}{\lambda_0}. \tag{6}$$

According to this equation a transverse magnetic field produces an increase of resistance unless $d\lambda$ is an increase in free path sufficiently large to make $d\lambda/\lambda_0$ greater than $\omega^2 T^2$.

If we subtract equation (3) from equation (6) we get

$$\frac{dR}{R} - \frac{\delta R}{R} = H^2 \left(\frac{e}{m}\right)^2 \frac{\lambda_0^2}{V^2} - \frac{d\lambda}{\lambda_0} + \frac{\delta\lambda}{\lambda_0}. \tag{7}$$

In the general case we may not set the last two terms of this equation equal to each other because of the crystalline nature of the specimens under examination. It is usual to experiment with wires which have been pulled through draw-plates, and in specimens of this kind it is possible for the crystal structure to differ along different directions in the metal. A magnetic field perpendicular to the wire, therefore, might produce an effect on molecular arrangement which is different from the effect of a field parallel to the wire. In case we have a substance which is magnetically isotropic the last two terms of the equation above become equal and we get

$$\frac{dR}{R} - \frac{\delta R}{R} = H^2 \left(\frac{e}{m}\right)^2 \frac{\lambda_0^2}{V^2}. \tag{8}$$

This equation has the advantage of being free from terms involving

[1] "Electricity in Gases."

unknown molecular changes and hence may be used for determining T. If we take the electrical conductivity of a metal to be given by[1]

$$\sigma = \frac{2}{3} n \frac{e^2}{m} \frac{\lambda_0}{V},$$

where n is the number of electrons per unit volume, we get

$$\frac{dR}{R} - \frac{\delta R}{R} = \frac{9}{4} H^2 \frac{\sigma^2}{n^2 e^2}. \tag{9}$$

According to this equation a transverse magnetic field should always produce a greater increase of resistance than a longitudinal field.

EXPERIMENTS.

It has been observed[2] that some forms of graphite suffer a large resistance change while other forms are apparently unaffected by a magnetic field. No systematic study seems to have been made of the reasons for these variations, and inasmuch as the variations are quite large it seemed as if a knowledge of the causes of these variations would prove illuminating from the standpoint of the theory. Furthermore, powdered graphite, composed of small crystals, may be compressed into bars, and these bars are similar to most metals in that they are composed of crystal agglomerations.

The apparatus which was used in measuring the resistance of the specimens was a Wheatstone bridge with balancing shunts so arranged that changing one of these shunt resistances by a large amount compensated for a very small change in the resistance of the specimen. A Leeds and Northrup high sensibility galvanometer of resistance 13 ohms was used, and the apparatus was sufficiently sensitive in most cases to measure a value of dR/R as small as 3×10^{-5}. The specimen under examination was suspended by means of a wooden clamp or an ebonite rod between the poles of a Weiss electromagnet in such a way that a simple rotation of the magnet sufficed to change a transverse into a longitudinal field without disturbing the specimen. This arrangement was found to be of distinct advantage, since the moving or jarring of the specimen was found under some conditions to alter its resistance. The experiments were performed at room temperature (about 26° C.) and in order to shield the specimen from air currents thick pads of hair-felt were set up around it between the magnet poles, which were 2.3 cm. apart. The bridge current was allowed to flow for a sufficient length of time before taking

[1] Swann, Phil. Mag., March, p. 441, 1914.

[2] D. E. Roberts, Konink. Akad. Wetensch. Amsterdam, Proc. 15, p. 148, 1912. G. E. Washburn, Ann. d. Phys., 48, 2, p. 236, 1915.

measurements to let the specimen come into temperature equilibrium with its surroundings. The resistance of the wires leading to the specimen was carefully measured and allowed for in the computations. These leads, however, were of copper and they had a low resistance, so that the effect of the magnetic field on the resistance of these leads was negligible.

Several forms of graphite were used in preparing the specimens for examination. The first group of experiments was made on the graphite of ordinary pencils, electric light carbons, and rods constructed for lubricating purposes. In these materials the pure graphite is mixed with " binder " of some sort—usually clay in the case of pencils—so that the resulting mixture is quite hard and brittle. In all these substances the effect of a magnetic field was quite small. Fig. 1 shows the relation between H and the resistance change for a rod of soft graphite made by the Dixon Crucible Co. for lubricating purposes. These curves are typical of other curves obtained for other materials of this class. It will be observed that the transverse effect is greater than the longitudinal effect for a given value of H. The experiments seemed to indicate that in soft specimens, presumably containing a small amount of binder, the magnetic field produces a greater effect than in the hard specimens.

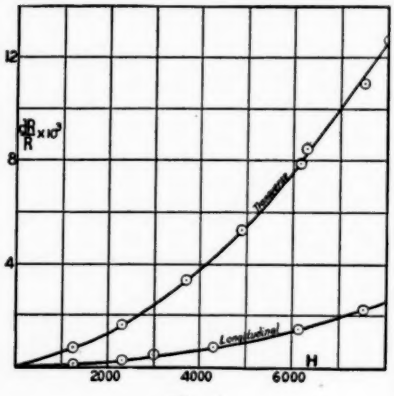

Fig. 1.

The next group of experiments was made on graphite powder compressed into sticks and bars. The following methods of compressing the powder were used:

(*a*) Hand compression between brass electrodes in small bore glass tubes. The electrodes were held by a screw arrangement in tight contact with the graphite and the graphite was not removed from the tube during the experiments.

(*b*) Compression of the powder by means of a hydraulic press into grooves in an ebonite block. The groove for each specimen consisted of a single straight shallow trough connecting two deep holes in the face of the block. Copper wires coiled in these holes were led out to serve as terminals. This block was placed face upwards at the bottom of a hollow iron cylinder, graphite poured into the cylinder, and pressure applied. When the block was removed and the excess graphite scraped off, the groove was found to contain a bar of graphite formed by pressure applied in a direction perpendicular to its length.

(*c*) Compression into the form of thin plates by means of the hydraulic press. The same method was used as in (*b*) except that no grooves were cut in the ebonite block. The thin brittle plates which were formed were cut into bars with a razor blade, the ends copper plated, and copper leads were soldered to the specimen.

Two different grades of powder were used—the visible difference between the two grades being that one consisted of larger crystals than the other. It was evident upon inspection of the specimens prepared in the three ways described above that the result was not even approximately an isotropic material. The bars exhibited cleavage planes perpendicular to the direction in which pressure had been applied. We should, however, expect a result of this nature. The crystals of the powder were like very small thin plates, hence pressure applied to such an aggregation might be expected to produce an anisotropic effect by forcing the crystals to turn their large plane faces perpendicular to the direction in which pressure had been applied while the specimen was being made up. When two test specimens were made up according to methods (*b*) and (*c*) respectively, it was found that a magnetic field produced the greater effect on the sample made by method (*c*). Since this method produces a much greater compression of the graphite powder than the other two methods it appears that high compression tends to increase the resistance change in a magnetic field. However, it was possible to change by hand the compression of samples made by method (*a*) until the resistance was halved, without producing an appreciable increase in the effect of a magnetic field. We must conclude, therefore, that a high degree of compression is necessary in order to produce much of an effect.

In Fig. 2 the results obtained with the fine power are expressed in graphical form; in Fig. 3 some of the corresponding results for the coarse powder are plotted. The letter on each curve specifies the method (described above) used in preparing the specimen. Circles are used to specify all results obtained when the magnetic field was perpendicular to the direction of the current; small crosses are used when H was parallel to the current. When the magnetic field was parallel to the direction in which pressure had been applied during the process of making the specimen the symbol $\parallel$ is written on the curve; when H was perpendicular to this direction the symbol $\perp$ is used. The immediate conclusions which we may draw from these curves are as follows:

1. Increasing the size of the graphite particles increases the effect of the magnetic field on the resistance; for the curves of Fig. 3 are all higher than the corresponding curves of Fig. 2.

2. The direction of the magnetic field with respect to the crystal structure in graphite is of greater importance than its direction with respect to the electric current. For in (*b*) of Fig. 2 the effect of a transverse field is only a small amount greater than the effect of a longitudinal field, provided the field is kept perpendicular to the direction of compres-

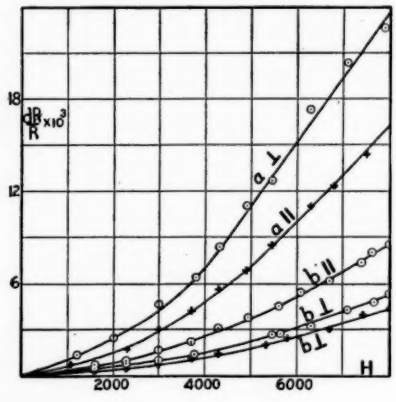

Fig. 2.

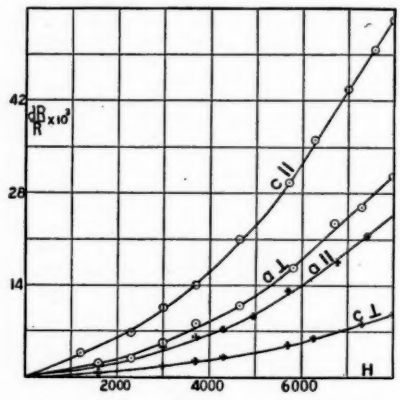

Fig. 3.

sion. Turning the field parallel to the direction of compression increases the effect of the transverse field.

3. If the effect of crystal structure in the graphite is eliminated we secure an effect in a transverse field which is greater than in a longitudinal field. This conclusion follows if we assume magnetic isotropy in all directions perpendicular to the direction of compression. We are then able to compare the two lowest curves of Fig. 2 with each other on the assumption that the influence of crystal structure is the same for each curve. In Fig. 2 the transverse field of 8,000 gausses gives a value of dR/R, which is greater by about 8×10^{-4} than that of the longitudinal field.

In order to test these conclusions more completely an experiment was performed on some fairly large crystals of graphite. These crystals consisted of laminated fragments embedded in a calcareous material which could be broken with comparative ease into small pieces. After some difficulty a satisfactory specimen of graphite was secured. The laminations of this specimen were parallel to its length, were very numerous and closely pressed together, and appeared to be very free from impurities. It was cut into the form of a bar, the ends copper plated, and copper terminals were soldered to these ends. The dimensions of the specimen were roughly $0.3 \times 0.4 \times 5.0$ mm., and its resistance was 0.08 ohm. The experimental results obtained with this speci-

men are plotted in Fig. 4. Here the symbols $\perp$ and $\parallel$ mean that H is respectively perpendicular and parallel to the normal to the laminæ of the specimen. These curves are similar to those of Figs. 2 and 3 except for the relative magnitudes of the effects. Where a single crystal or a single group of crystals is used it is evident that the magnitude of the resistance change is greatly increased. In short, we may consider that the experiments with the large crystal group corroborate conclusions (1) and (2) above. Regarding conclusion (3) it is to be observed that of the two lowest curves of Fig. 3 that for the transverse field is the higher. Another sample of crystalline graphite tested out very carefully in this respect gave similar results to those plotted in Fig. 3. If we make the assumption that the crystal is magnetically isotropic for all directions in the plane of the laminations then we may assert that a transverse magnetic field produces a greater effect than a longitudinal field when the structural changes produced by H are the same in both cases.

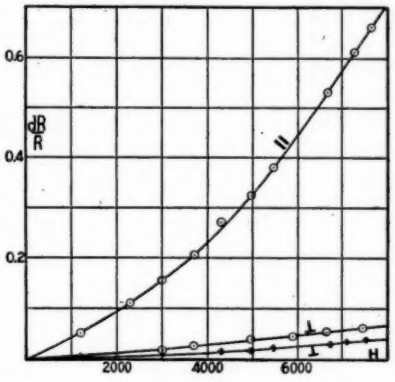

Fig. 4.

The general conclusions, however, which are to be drawn from the above study of a group of small crystals indicate very clearly that we must inquire carefully into the crystalline nature of metals before attempting to interpret experiments which are made on metals. The point of immediate interest is to decide whether or not a wire of given material is magnetically isotropic as regards directions parallel and perpendicular to its length. Ordinary magnetic measurements are difficult to make with sufficient accuracy in this case, and furthermore, we cannot be sure that isotropy as regards magnetic permeability, for example, will guarantee isotropy as regards resistance change in a magnetic field. A way in which we may gain information, however, is as follows. If a thin section of metal is cut from a cylindrical bar of cast metal (the cut being perpendicular to the length of the bar) and then hammered out into a thin plate we may suppose from principles of symmetry that for all directions in the plane of the plate we will have magnetic isotropy.

A thin rectangular plate of cadmium was prepared in this way and a series of parallel cuts made across it. These cuts did not completely traverse the plate, but were so made that four very thin strips of metal

were secured, each about two centimeters long and half a millimeter wide, lying side by side and joined in series. When experiments were made on this specimen it was found that a transverse magnetic field of 7,550 gausses produced the same increase of resistance whether parallel or perpendicular to the surface of the strips. A difference in dR/R as small as 3×10^{-5} could have been detected in the two cases. Keeping the magnetic field in the plane of the strips but rotating it so as to compare the longitudinal and transverse effects, it was found that the transverse field produced the greater increase of resistance. The average of two trials gave

$$\frac{dR}{R} - \frac{\delta R}{R} = 9 \times 10^{-5}$$

with $H = 7{,}550$. This result agrees as well as could be expected with previous experiments[1] made on a different sample of cadmium in the form of wire. We may conclude then, that the cadmium strips were magnetically isotropic with respect to resistance change. If strips of metal hammered out in this fashion are magnetically isotropic it is probably safe to assume that cadmium wires are similarly isotropic as regards resistance change.

It was difficult to test other non-ferromagnetic metals than cadmium in this way for magnetic isotropy because those metals which are easily worked with mechanically have a small resistance change in a magnetic field, and hence require a more sensitive apparatus than the one available. In a previous paper[2] the writer has measured both the transverse and longitudinal effects in wires made of different metals. In all cases the transverse effect was greater than the longitudinal as we should expect from equation (9). If we assume that the other metals are like cadmium in being magnetically isotropic we may apply equations (8) and (9) to these substances and calculate the mean free period of the electrons and the number of electrons per unit volume of metal. The following table gives the results of the calculations, data being taken from the previous paper except in the case of bismuth and graphite. In the calculations it is assumed that $e = 1.6 \times 10^{-20}$ e.m.u., $e/m = 1.7 \times 10^{7}$, and $H = 8{,}000$ gausses.

Patterson[3] obtained values of n somewhat different from those given above but in his equation no account was taken of the longitudinal effect or of the effect of H on molecular structure. Other methods of determining n are based upon Drude's theory of the optical properties of

[1] Phil. Mag., Dec., p. 900, 1911.

[2] Phil. Mag., Dec., p. 900, 1911; Dec., p. 813, 1912.

[3] Phil. Mag., 3, p. 643, 1902.

	σ(e.m.u.).	$\frac{dR}{R} - \frac{\delta R}{R}$.	T.	n.
Te[4]	[2]4.7 $\times 10^{-5}$	4.9 $\times 10^{-3}$	5.1$\times 10^{-13}$	5.0 $\times 10^{20}$
Bi[5]	[2]0.84	0.144	39.2	11.8 $\times 10^{18}$
PbS[6]	[3]0.042	1.15$\times 10^{-4}$	0.8	2.9 $\times 10^{19}$
Cd	[2]13.0	6.5 $\times 10^{-5}$	0.59	1.2 $\times 10^{22}$
Zn	[2]16.0	2.9 $\times 10^{-5}$	0.4	2.2 $\times 10^{22}$
Au	[2]41.0	0.3 $\times 10^{-5}$	0.12	18.8 $\times 10^{22}$
Graphite[7]	[3]0.036	0.024	11.4	1.74$\times 10^{18}$

[2] From Kaye and Laby.

[3] Baedeker, "Elektrischen Erscheinungungen in Metallischen Leitern."

[4] Cast in cylindrical form.

[5] The tests on bismuth were made using a Hartmann and Braun spiral. The magnetic field used in this case was 5,700 gausses.

[6] Natural crystal. Tests showed it to be magnetically isotropic.

[7] Large natural crystals of Fig. 4.

metals and upon the theory of thermionic emission. Experiments made by Spence[1] on the refractive index of metals lead him to give as probable values of n for gold the number 3.3×10^{22}, for silver 3.7×10^{22}, and for platinum a number less than 8.1×10^{22}. From measurements on the thermionic emission of hot metals H. A. Wilson[2] deduces for platinum a value of $n = 1.5 \times 10^{22}$. These results are of the same order of magnitude as those tabulated above.

It is of interest to consider how equations (3) and (6) may be applied to the experiments. In the case of all the substances listed in the above table we must consider the term $\delta\lambda/\lambda_0$ to be very important, for in all these materials the change of resistance in a longitudinal magnetic field is comparable with the change in a transverse field. We are thus led to believe that in equation (6) the term $d\lambda/\lambda_0$ must account for a large part of the resistance change. Furthermore $\delta\lambda$ must represent a decrease in free path, otherwise a longitudinal magnetic field would decrease the resistance of the conductor—a result contrary to experiment. It is to be hoped that a more complete understanding of this term, $\delta\lambda$, will be reached by further experiment.

Conclusions.

1. The theory of the effect of magnetic fields on resistance has been developed to cover the cases of longitudinal and transverse magnetic fields. The theory leads us to expect a greater transverse than longitudinal effect. All experiments on non-ferromagnetic metals appear to be in agreement with this conclusion. In order to make definite tests,

[1] Phys. Rev., 28, p. 337, 1909.

[2] "Electrical Properties of Flames."

however, we are obliged to consider carefully the crystalline structure of the metals examined.

2. With this fact in mind experiments have been made on crystalline graphite in different forms. In compressed graphite powder the effect of a magnetic field on the resistance is greater in coarse powder than in fine. This fact implies that in a substance composed of large crystals the effect is greater than when it is composed of small crystals. It is probable that this effect of crystalline structure explains why bismuth, which in ordinary form suffers a large increase of resistance in a magnetic field, is very little affected when in the form of fine powder produced by chemical reduction of a bismuth salt.

There is a greater effect in large crystals of graphite than has so far been discovered in any substance which is metallically conducting. From the study of graphite it is to be concluded that crystal structure is an important factor to consider when experimenting on the conductivity of any substance in a magnetic field. By making up specimens in the proper way it is possible to compare the longitudinal and transverse effects experimentally without any trouble arising from differences in crystalline structure.

3. From experiments on a specially prepared sample of cadmium it is to be concluded that comparisons of the longitudinal and transverse effects may be made legitimately using an ordinary wire as the specimen. Assuming that wires of other metals are like cadmium in this respect calculations have been made for several metals of the number of electrons per unit volume and of the free period of the electrons.

4. The theory affords a satisfactory explanation of the behavior of all para- and diamagnetic substances which have so far been examined.

The writer is indebted to Mr. N. S. Diamant, of the department of engineering, for aid in securing graphite samples; also to the Dixon Crucible Co. for kindly supplying crystals of graphite in a natural condition.

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THE TRUE TEMPERATURE SCALE OF TUNGSTEN AND ITS EMISSIVE POWERS AT INCANDESCENT TEMPERATURES.

BY A. G. WORTHING.

INTRODUCTION.

A TRUE temperature scale for tungsten at incandescent temperatures based on sound principles was first obtained by Pirani[1] when he bent a tungsten ribbon back and forth so as to obtain a cavity largely surrounded by it, which was raised to incandescence in the ordinary manner by a heating current. He concluded that the radiation coming from the interior of the cavity was black body radiation. Thus he was able to express, with the aid of a Holborn-Kurlbaum optical pyrometer, the relation between the brightness temperature[2] of the natural tungsten radiation and the true temperature of the tungsten, that is the brightness temperature of the radiation from the cavity. An emissive power relation follows simply. He concluded that for $\lambda = 0.64\mu$ the emissive power was constant and equal to about 0.485. However the uncertainty as to this result was rather large, being stated as $7\frac{1}{2}$ per cent.

Soon afterwards Mendenhall and Forsythe[3] used a narrow V-shaped trough and in a similar manner obtained a temperature scale which involved emissive powers increasing with temperature from 0.45 at 1100° C. to 0.66 at 2900° C. Unfortunately their results were subject to considerable error because the two strips separated at the apex of the V at high temperatures.

Two other scales based on somewhat similar pyrometer measurements have been developed by Pirani and Meyer[4] and by Langmuir.[5] In both instances the brightness of the interior of a closely wound tungsten helix was compared with the brightness of the exterior. Pirani and Meyer's results indicate that the emissive power at the wave-length 0.532μ, which

[1] Phys. Zeit., 13, p. 753, 1912.

[2] Heretofore the term "black body temperature" has been used to designate this quantity. The reasons for abandoning this term in favor of "brightness temperature" are fully stated by Hyde, Cady and Forsythe in the paper following this.

[3] Astrophys. Jour., 37, p. 380, 1913.

[4] Elektrot. u. Masch., 33, pp. 397 and 414, 1915.

[5] PHYS. REV., II., 6, p. 138, 1915.

seems to have been used by them, is constant with temperature and equal to 0.44. Langmuir concluded that the emissive powers at 0.667μ and 0.537μ were independent of the temperature and equal respectively to 0.465 and 0.485. Later[1] he concluded 0.46 to be the most probable value for the wave-length 0.664μ. The difference between the two scales for 0.532μ and 0.537μ is considerable. The earlier temperature scales which these experimenters had arrived at, and to some extent, the later scales just reported, have been founded on or tempered by the results of Holborn and Henning[2] who concluded that the emissive powers of silver, gold, platinum, and palladium in the visible spectrum were independent of the temperature.

Shackelford[3] working in this laboratory, using helical coils of varying pitch, and extrapolating for the case of a closed helix obtained, for a red brightness temperature of 2300° K. (true temperature of 2530° K.), 0.445 as the emissive power at 0.656μ and 0.465 at 0.493μ. At a temperature about 400° lower values slightly larger were obtained. Others have measured the emissive power of tungsten at some one temperature. These are well summarized by Burgess and Waltenberg[4] who obtained 0.39 at 2020° K. for 0.65μ. Considering further only the later values as the more probable, there are in addition Coblentz's[5] 0.474 at 0.65μ, Wartenburg's[6] 0.51 at 0.65μ and Littleton's[7] 0.545 at 0.589μ, all of which refer to room temperature. Other temperature scales depending on the fact that the luminous flux from a tungsten filament may be matched in color with that from a black body are discussed in the following paper by Hyde, Cady and Forsythe.

METHOD AND APPARATUS.

General Procedure.—In the present work, except for the measurements at room temperature, as will appear later, long tubular filament of tungsten with small holes penetrating the side walls at various places have been made use of. In general terms, the procedure has consisted of determining with an optical pyrometer the ratio of the brightness of the filament surface adjacent to a hole, in a region suitably chosen from the standpoint of constancy of temperature, to the brightness of the hole, when the filament was heated to incandescence in a vacuum or in an

[1] PHYS. REV., II., 7, p. 302, 1916.
[2] Berl. Ber., p. 311, 1905.
[3] Jour. Frank. Inst., 180, p. 619, 1915, and PHYS. REV., II., 8, p. 470, 1916.
[4] Bull. Bur. of Stds., 11, p. 591, 1915.
[5] Bull. Bur. of Stds., 7, p. 197, 1911.
[6] Verh. der Deut. Phys. Gesell., 12, p. 105, 1910.
[7] PHYS. REV., 35, p. 306, 1912.

atmosphere chemically inert. On the assumption that the radiation from the hole is black and that there is a negligibly small difference of temperature between the interior and the surface, such a ratio represents an emissive power for a wave-length depending on the light transmitted by the pyrometer glass screen, and for a temperature corresponding to that of the radiation from the hole. This latter temperature was obtained in the standard manner with the aid of Wien's law by comparing the black radiation with that from a calibrated black body of the ordinary type at the palladium point. As already noted a brightness temperature true temperature relation follows simply. The assumptions made and the corrections for the lack of their fulfillment are considered in detail in the section on Corrections and Errors.

Preparation of Filaments.—The filaments themselves were formed according to the commercial method common about five years ago by squirting a paste of tungsten powder held together by a binder through a die, in the present case one with an annular opening. Shortly after the actual squirting, when the tubes were of the proper consistency, small holes were pierced in many places through the walls. Following the usual drying and heating, the filaments were mounted in lamp bulbs. At this stage the filaments used had external and internal diameters of about 1.3 mm. and 0.8 mm. respectively. The holes through the walls were nearly circular and of two sizes, approximately 0.09 mm. and 0.12 mm. in diameter.

Much difficulty has been experienced in mounting these filaments in lamp bulbs because of the extremely large currents, 100 amperes being the maximum, which were required. It was considered impracticable, after several failures, to use soft glass bulbs. Many successful lamps using hard glass have been made, but the bulbs of those made at first contained so many bad streaks that it was often impossible even with selected bulbs, to obtain observations on more than one or two portions of the filaments. The later lamps have been fairly satisfactory in this respect, however. Some of the bulbs have been evacuated, but most of the data reported have been on filaments immersed in a gas, usually argon.

Apparatus.—The optical pyrometer used was of the Holborn-Kurlbaum or Morse type such as has been used quite commonly in this laboratory.[1] As pyrometer screens a red glass, Jena F–4512, and a blue uviol glass in single and double thicknesses have been used. As will be shown later, the changes in thickness and the lack of monochromatism were readily corrected for, so that the final results may be considered as applying

[1] PHYS. REV., II, 4, p. 163, 1914.

strictly to two definite wave-lengths, viz., 0.665μ and 0.467μ. Photographs are reproduced in Fig. 1, which show considerably magnified what is seen when the pyrometer filament appears somewhat less bright than the hole but brighter than the adjacent surface for each of the two sizes of holes used.

PRELIMINARY TESTS.

As is usually the case, these tests are preliminary from the standpoint of character rather than from the standpoint of time of performance. They represent tests which were essential, before any reliability could be placed upon the main experimental data obtained.

Uncertainty Due to Smallness of Holes.—Fig. 1 suggests that, due to the smallness of the holes, from physiological and psychological grounds, one might be expected to make erroneous judgments, thus vitiating the results. In order to test this, pyrometer settings were made on an extended luminous background such as was described by Lorenz,[1] first when viewed through a fine needle hole in an opaque screen just in front of the background, then when viewed without the opaque screen. With the fine needle hole of approximately the same size as the holes in the filaments used, no systematic differences in the pyrometer readings, which depended upon the presence or absence of the opaque screens, were noticeable. The same conclusion as to freedom from error on this account was borne out by the results obtained with changes in the magnification under workable conditions.

Distortion in Temperature Distribution Due to Presence of Holes.—Unquestionably the presence of such holes as were pierced in the walls of the filament caused variations in the temperature distribution in their neighborhood. Many times tests for such changes including settings as close as 0.02 mm. to the edge of the hole were made, but in no case was such an effect detectable.

Constancy of Temperature of the Surface on a Given Circumference.—Broken filaments showed in general that the inner and the outer surfaces of the filament wall were not coaxial, but that the maximum and minimum thicknesses of wall at any one cross-section varied as much as 7 to 5. Attempts to determine the effect of this mathematically have been unsuccessful. Pyrometer settings at various positions around a circumference revealed no certain differences at temperatures above 1500° K. Data at lower temperatures were not conclusive. It is further believed that in the average any effect of this type would be eliminated.

Effects of Preliminary Heating.—The first results obtained showed that

[1] Elec. World, 61, p. 932, 1913.

Fig. 1.

Photographs showing the pyrometer filament projected against the hole and the surface as a background for each of the two sizes of holes used.

A. G. WORTHING.

a gradual change was taking place. With continued operation the values obtained for the emissive powers gradually decreased, the total change amounting to as much as 7 or 8 per cent. of the quantity measured. It was soon found that the filament could quickly be brought fairly close to its final steady state by a preliminary heating for a short time at a high temperature. In subsequent work such preliminary heat treatment at about 2800° K. was always given, usually previous to the completion of the lamp, while it was still connected with the evacuating pump. That such a temperature was reached was assured by a method common in this laboratory, in which there were compared the colors of the two shadows of a pencil or some slender opaque object on a piece of white paper as produced by the lamp being tested and an ordinary commercial 100-watt gas-filled tungsten lamp. It is necessary to have the two shadows about equally bright. In the heat treatment given, the temperatures reached were always such as to indicate that the color of the light from the lamp tested was noticeably bluer than that from the commercial lamp.

Effects of Lack of Surface Polish.—The accidental short-circuiting of a resistance, which caused the filament being studied to melt at a certain cross-section, was apparently also the means whereby the surface was partially polished. The subsequent tests with this filament seemed to give lower values for the emissive power than were obtained previously. Later tests on polished and ordinary unpolished filaments showed this effect to be real and to account for differences which may amount to two per cent. in the emissive power. By variations in the process of preparing squirted tungsten filaments, filaments having various surface appearances may be obtained. For definiteness of results the need of specifying the surface-character cannot be overstated. In illustration of this it is sufficient to say that the writer has in his possession filaments, the structure of which is such that the surface has a large diffuse reflectivity. Emissive powers for these filaments as ordinarily measured are of the order of 50 per cent. greater than those for polished filaments. Because of these considerations, the determinations of emissive power to be reported have been confined to polished or fairly well polished material.

Effective Wave-lengths of Blue Glass Screens.—A further preliminary test consisted of determining the effective wave-lengths of the blue uviol glass. The red glass had previously been studied by Hyde, Cady and Forsythe.[1] As defined by them, the effective wave-length for a screen for a definite temperature change in the source viewed is that wave-length for which

[1] Astrophys. Jour., 42, p. 294, 1915.

the relative change in monochromatic brightness is equal to the relative change in total brightness for the luminous flux transmitted by the screen. Following a method much as reported previously[1] and as more fully outlined by Langmuir[2] the effective wave-lengths of a single and of a double thickness of the blue glass have been determined for different temperature intervals of black body radiation. By platting the logs of the blue brightnesses as a function of the logs of the red brightnesses for ranges of 100 to 1 of the latter for black body radiation, a very good straight line relation was found. The slopes, using one or two thicknesses of the blue glass against two of the red glass, were respectively 0.745 and 0.706 indicating that if for black radiation for a given temperature interval, the effective wave-length of a screen composed of the two red glasses is, say, 0.665μ, the corresponding effective wave-lengths for the blue glass are respectively 0.495μ and 0.469μ. Unfortunately, the method is insensitive in showing variations in effective wave-lengths. All that may be said is that the effective wave-lengths thus determined are average values. The variations with temperature, for the blue glass using the method described by Hyde, Cady and Forsythe, were found to be fully five times as great relatively as those they found for the red glass. Further considerations due to the lack of monochromatism in the transmission of the screens will be considered in the next subdivision.

CORRECTIONS AND ERRORS.

Difference in Temperature between the Inner and Outer Surfaces.—A simple formula given in a paper by Angell[3] expresses this difference in terms of the thermal conductivity and of ordinary measurable quantities. Letting

r_0 = external radius of the hollow filament,
r_1 = internal radius of the hollow filament,
i = current density,
ρ = resistivity,
k = thermal conductivity,
T = temperature,
ΔT = increase in temperature in passing from the external to the internal surface,
E = radiation intensity,
B = brightness,
B_λ = brightness ordinate at λ,

[1] Astrophys. Jour., 36, p. 348, 1912.
[2] PHYS. REV., II., 6, p. 146, 1915.
[3] PHYS. REV., II., 4, p. 535, 1914.

C_2 = constant in Wien's equation,
ϵ_0 = observed emissive power,
ϵ = corrected emissive power,
subscript (v) refer to filament in vacuo,
subscript (ar) refer to filament in argon,
we have

$$\Delta T = \frac{i^2\rho}{2k}\left(r_i ln\frac{r_0}{r_i} - \frac{r_0^2 - r_i^2}{2}\right).$$

For the filaments used for the greater part of the work r_0 and r_i were respectively 0.66 mm. and 0.38 mm. Taking account of the simply derived relation

$$E = \frac{r_0^2 - r_i^2}{2r_0} i^2\rho,$$

we have

$$\Delta T = \frac{E}{k} \times (0.040 \text{ cm.}).$$

The effect of this ΔT on observed emissive powers is seen when one computes with the aid of Wien's equation, the relative increased brightness of the hole resulting from the existence of this ΔT. Thus

$$\frac{1}{B_\lambda}\frac{\partial B_\lambda}{\partial T}\Delta T = \frac{C_2}{\lambda T^2}\Delta T.$$

Evidently also

$$\frac{\epsilon - \epsilon_0}{\epsilon_0} = \frac{C_2}{\lambda T^2}\Delta T.$$

To obtain values applicable to the gas-immersed filament, it is only necessary to multiply the correction here found by the ratio of the square of the current when thus immersed to the corresponding value for the filament in a vacuum. Values of k and E for tungsten taken from a table appearing in a later subdivision lead to the results given in Table I. The effects of these corrections will be shown later.

Lack of Blackness in Radiation from the Hole.—There are three factors tending toward departure from perfect blackness in this radiation, (1) the presence of the small hole for observing, (2) the existence of a temperature gradient along the tube, and (3) the presence of possible crystal surfaces on the inner surface of the tube.

In connection with the first factor, it is easy to compute the departure from blackness on the supposition of a long tube of uniform temperature with a perfectly matt interior surface. For the smaller of the two sizes of holes specified, and with an assumed emissive power of 0.45, it follows that the radiation will deviate from blackness quantitatively by about

TABLE I.

Emissive-power Corrections for the Temperature Difference between the Internal and External Surfaces of the Filaments Used.

T.	ΔT_v.	$\lambda=0.665\mu$.			$\lambda=0.467\mu$.		
		$\frac{T}{B_\lambda}\frac{\partial B_\lambda}{\partial T}$.	$\left(\frac{\epsilon-\epsilon_0}{\epsilon_0}\right)_v$.	$\left(\frac{\epsilon-\epsilon_0}{\epsilon_0}\right)_{ar}$.	$\frac{T}{B_\lambda}\frac{\partial B_\lambda}{\partial T}$.	$\left(\frac{\epsilon-\epsilon_0}{\epsilon_0}\right)_v$.	$\left(\frac{\epsilon-\epsilon_0}{\epsilon_0}\right)_{ar}$.
1500° K.	0.22°	14.4	.002	.004	20.5	.003	.006
2300	1.54	9.4	.006	.007	13.4	.009	.010
3100	5.6	7.0	.012	.013	9.9	.018	.019

0.1 per cent.; for the larger size holes it will be about two times this or 0.2 per cent.

Quantitative computations regarding the effects of the second factor in producing a departure from blackness are difficult. Measurements have been made almost entirely on portions of the filament where the temperature was constant to within a few degrees over lengths on each side for distances of at least five times the internal diameter of the tube. Moreover, measurements intentionally taken where a noticeable temperature gradient existed did not yield results noticeably different. Errors from this source will be more noticeable at the low temperatures than at the high temperatures, because the cooling effects of the supports and leading-in wires are confined to shorter lengths of the filament at the higher temperatures, these lengths being inversely proportional to the heating currents. Errors from this source are probably very small.

The third factor tending away from blackness was a matter of some concern in connection with a certain filament, particularly following the short-circuiting of a resistance in series with it and the consequent melting of a portion of the filament as previously mentioned. Dark irregular patches were noticed within the holes. Later microscopic inspection of the filament showed the surface to be made up of comparatively large crystal surfaces. The accidental orientation of such crystals normal to the line of sight on the inner wall and in line with a hole were apparently the explanation of the dark patches mentioned. The occurrence of a large number of such surfaces oriented irregularly is of course equivalent to a matt surface, such as has been considered already. In the experimental work, by arbitrarily orienting the filament, the spots were eliminated from the field of view. Results with this filament were not noticeably different from those with other polished filaments.

Except in giving the lower values for the emissive power at a given temperature greater weight than the higher values, no correction for these departures have been made.

Lack of Monochromatism in the Light Used.—It is necessary to consider here to what wave-lengths to ascribe[1] the brightness temperature measurements made with the roughly monochromatic screens used in optical pyrometry, and how to correct the emissive power determinations made so that they will uniformly apply to a single wave-length.

Consider in this connection Fig. 2, in which in an exaggerated way curves α, β, γ and δ represent for a given filament at a temperature T, certain spectral brightness B_λ distributions related to the luminous flux transmitted through the pyrometer system including the colored-glass screen at the eyepiece. Let α refer to the black body radiation at

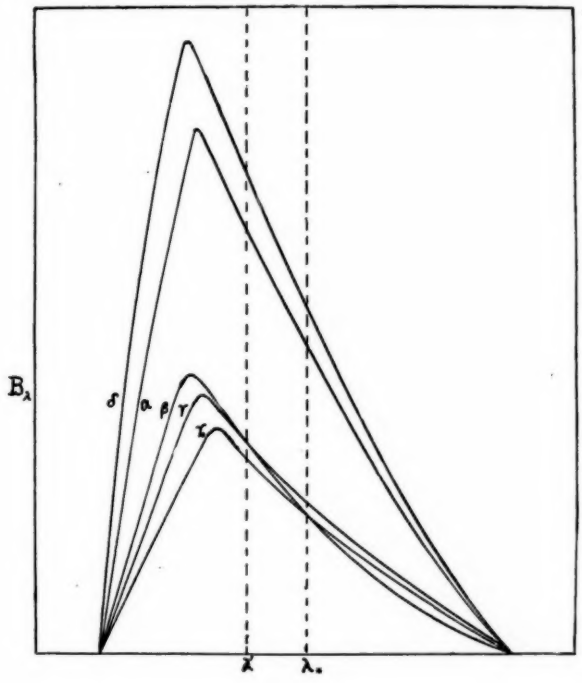

Fig. 2.

A diagram showing various spectral brightness distributions connected with tungsten filaments such as used, which are helpful in determining the wave-length to which to ascribe brightness temperature measurements.

the temperature T coming from a hole in the filament wall; β the natural tungsten radiation arising from the adjacent external surface; γ the radiation from a black body having the temperature S, the measured brightness temperature of the natural tungsten radiation; and δ that black body radiation whose relative brightness distribution is the same as that given by β. These diagrammatic distributions assume the pos-

[1] The writer is in part indebted to his colleague W. E. Forsythe for this development.

sibility of color matching the tungsten radiation with black body radiation. Thus curve δ is, according to Hyde, Cady and Forsythe,[1] the brightness distribution of a black body at a temperature given by the color temperature of the natural radiation. Evidently from the definition of brightness temperature the areas included under curves β and γ are equal. It is also evident that only at the wave-length λ' is the brightness temperature of the natural radiation equal to S, being progressively less than S as the wave-length is increased beyond λ' and progressively greater than S as the wave-length is decreased below λ'.

Representing by ${}_{\alpha}B_{\lambda}$, ${}_{\beta}B_{\lambda}$, etc., values of B_{λ} corresponding to curves α, β, etc. and by ${}_{\alpha}B$, etc., the total brightnesses $\int_0^{\infty} {}_{\alpha}B_{\lambda} d\lambda$, etc., we then have

$$ {}_{\beta}B = {}_{\gamma}B, $$

$$ \frac{{}_{\beta}B_{\lambda}}{{}_{\delta}B_{\lambda}} = \frac{{}_{\beta}B}{{}_{\delta}B} = \frac{{}_{\gamma}B}{{}_{\delta}B} = \frac{{}_{\gamma}B_{\lambda'}}{{}_{\delta}B_{\lambda'}}, $$

where in the first member λ, of course, refers to any wave-length within the range concerned. The last of the above equations according to Hyde, Cady and Forsythe[2] is also the defining equation of the effective wave-length of the pyrometer screen for black body radiation for the temperature interval given by curves γ and δ. It follows therefore that the wave-length λ' to which the brightness temperature S is to be ascribed is the effective wave-length of the screen for black radiation in going from the brightness temperature of the tungsten to its color temperature. In the writer's work λ' for tungsten has varied from 0.6662μ at 1600° K. true temperature to 0.6628μ for 3200° K.

Having once determined λ', the method of determining S_0, the brightness temperature which shall correspond to some common wave-length λ_0 arbitrarily chosen, is simple. It consists in finding the temperature of a black body corresponding to γ_0 (Fig. 2). γ_0 must evidently intersect β at λ_0. The application of Wien's law to a change in which

$$ \frac{{}_{\beta}B_{\lambda_0}}{{}_{\beta}B_{\lambda'}} = \frac{{}_{\gamma_0}B_{\lambda_0}}{{}_{\gamma}B_{\lambda'}} = \frac{{}_{\delta}B_{\lambda_0}}{{}_{\delta}B_{\lambda'}}, $$

gives the result desired. Choosing λ_0 as 0.665μ means in the writer's work that the values of $S_0 - S$ for tungsten for the red light are respectively $+ 0.2°$ and $- 1.4°$ at true temperatures 1600° K. and 3200° K. The corrections for the blue uviol screen are somewhat greater.

In a similar way the wave-length to which to ascribe the emissive

[1] See following paper.

[2] Astrophys. Jour., 42, p. 294, 1915.

power measurements, may be determined. Imagine another spectral brightness distribution curve added to the somewhat complicated figure, which shall enclose underneath it an area equal to that enclosed by β, and which shall bear the same relation to α that β does to δ. Call this curve β'. The ratio of its ordinates to that of α will everywhere be equal to the measured emissive power. It will cross the curve β at some wave-length λ''. Evidently at this wave-length only is the ratio of the ordinate of β to that of α equal to the measured emissive power. Hence strictly the emissive power measured should be ascribed to λ''. As in the case of λ' just described, λ'' may be shown to be the effective wave-length for the optical system in passing from distribution α to distribution δ. λ'' is slightly shorter than λ'. On considering later the change in emissive power in going from 0.665λ to 0.467λ together with color matching possibilities, it will be seen that the changes in the emissive power in going from λ'' to λ_0 are very small. In this work such corrections at 0.665μ were inappreciable, those at 0.467μ were just appreciable, as will appear later.

At temperatures below 1500° K. in the case of the red light and below 1700° K. in the case of the blue light, single thicknesses of pyrometer screens were used. The corrections to be applied according to the foregoing principles in order to obtain values to be expected if the regular double thicknesses has been usable, are appreciable but not large.

STANDARDIZATIONS.

Any expression of emissive power as a function of temperature necessarily implies a temperature scale which in turn is based on certain standardization points. In the preliminary notice of this paper[1] the temperature scale was based on 1336° K. and 1822° K. as the melting points of gold and palladium respectively. As shown by Hyde, Cady and Forsythe,[2] this with the assumption of Wien's law, which in its effects is indistinguishable in the visible spectrum from Planck's law, leads to a C_2 of $14460\mu \times$ deg. For reasons stated elsewhere[3] our laboratory has abandoned this scale and adopted that one based on 1336° K. as the melting point of gold and $14350\mu \times$ deg. as the value of C_2. This leads to 1828° K. as the melting point of palladium. The importance of stating these underlying bases of temperature scales when one is mentioned should be strongly emphasized. The temperature measurements in the present work were carried out with the aid of a large tungsten

[1] Jour. of Franklin Inst., 181, p. 417, 1916; PHYS. REV., II., 7, p. 497, 1916.

[2] Astrophys. Jour., 42, p. 300, 1915.

[3] Gen. Elec. Rev.—to appear soon.

filament lamp, which had been standardized as to brightness temperature at the palladium point by W. E. Forsythe, of this laboratory. The calibrations of the sectored disks used in conjunction with Wien's law in determining other brightness temperatures through comparisons with the standardized palladium point were made by a photometric method by F. E. Cady, also of this laboratory and are believed to be known in consequence of repeated determinations and checks with an accuracy of the order of 0.1 per cent.

The method of determining temperatures is given by the following equation

$$ln\, t = ln \frac{{}_0B_\lambda}{B_\lambda} = \frac{C_2}{\lambda}\left(\frac{1}{S} - \frac{1}{S_0}\right),$$

where λ = the effective wave-length,

S_0 = the brightness temperature of the standard (palladium point),

S = the brightness temperature being determined,

B_λ = the brightness ordinate at λ of the spectral brightness distribution curve,

${}_0B_\lambda$ = value of B_λ corresponding to S_0,

C_2 = constant in Wien's equation,

t = transmission of the sectored disk used.

RESULTS.

In the present paper only emissive powers in a direction normal to the surface or nearly so are considered. Values for other angles of emission may be computed with the aid of measured values of the deviation of the radiation from Lambert's cosine law.[1] The values there referred to, however, were obtained on unpolished material and must be so considered.

The experimental values obtained, except that those obtained with single thicknesses of pyrometer glass have been corrected as described so as to refer to double thicknesses, are platted in Fig. 3. Points indicated by difference symbols represent values, as per the accompanying caption, obtained with different filaments or possibly the same filament with the surface renewed by polishing. At room temperature a different procedure was followed. Here for the most part a polished filament previously used by Weniger and Pfund in infra-red measurements and discarded because of pits formed in use, and to some extent some mirror surfaces formed by melting carefully the larger portion of the ends of tungsten terminals in an arc lamp[2] were used. Both types of surfaces had previously the preliminary heat treatment already mentioned. The reflectivity was

[1] Astrophys. Jour., 36, p. 345, 1912.

[2] Langmuir, II., 6, p. 138, 1915; Luckey, PHYS. REV., II., 9, p. 132, 1917.

measured in the ordinary way, using the pyrometer apparatus as in the previous measurements. The measurements consisted of brightness determinations of a definite spot on a broad lamp filament, first when an mage of the filament was viewed directly, then when viewed reflected from the polished surface, there being, of course, identical optical paths

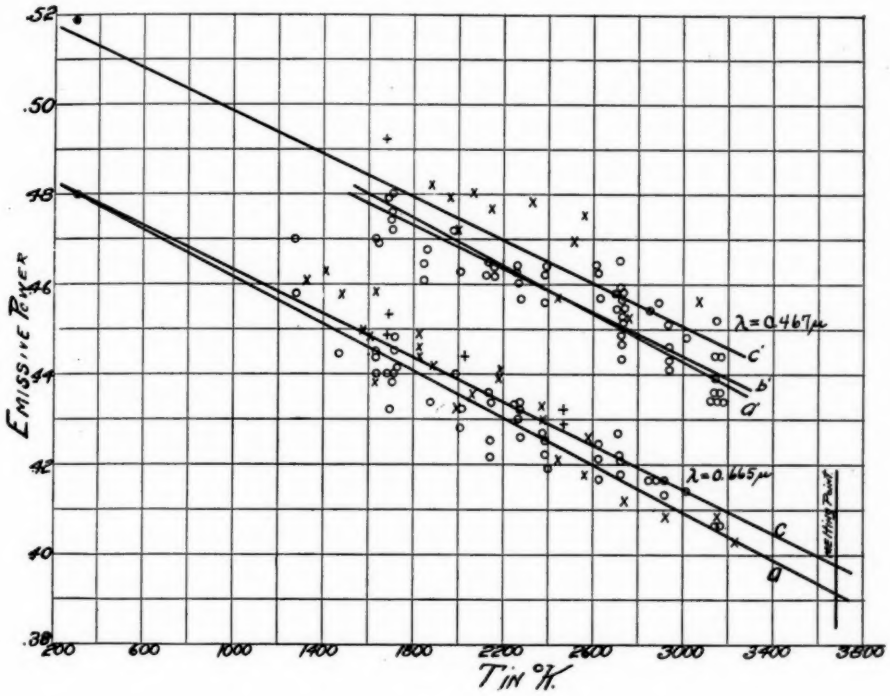

Fig. 3.

Emissive power results for tungsten as a function of the temperature at 0.665μ and 0.467μ.
×, values obtained on unpolished filament in much striated bulbs.
+, " " " polished filaments in much striated bulbs.
○, " " " polished filament in fairly clear bulbs.
●, " " at room temperature by reflection method.
a, *a'*, weighted curves for data obtained.
b', curve *a'* corrected for lack of monochromatism of the uviol glass.
c, *c'*, final curves containing corrections for differences in temperature between interior and exterior surfaces of the filaments.

in the two cases, except for the reflection from the polished surface. It was surprising to note how much the image of the broad lamp filament formed at the surface of the discarded polished filament was broken up by fissures, and at the same time that it was impossible to see any such fissures at all when the filament was self-luminous and viewed normally or nearly so. This indicates that for normal emission generally there

was no opportunity for blackening of the radiation. A consequence of this is that a rather rough polish of surface only is necessary for emission measurements normally on a self-luminous filament, a fact quite in contrast with the requirements for reflection measurements.

The earlier measurements were made on unpolished filaments in lamp bulbs which, as has been stated, distorted the images somewhat in almost every instance and did not always permit of the selection of holes entirely satisfactorily located from the standpoint of end cooling effects. In the later measurements a partially polished filament was used and the bulb was such as to permit of undistorted images. For these reasons in drawing the curves much emphasis has been given to the later measurements. Further, because errors due to lack of blackness in the radiation from the hole tend toward too high values of the emissive power, the lower values have been given greater weight than the higher values. The heavy lines a and a' in Fig. 3 show the weighted results. Line b' represents the curve obtained when corrections are made for lack of monochromatism in the pyrometer screen transmitting blue light. As stated previously in the case of the red light this correction is negligible. Curves c and c' represent the final emissive power curves in which corrections have been made for the difference in temperature between the interior and the surface of the filament.

When once an expression between temperature and emissive power for a substance is obtained, the use of Wien's equation enables one to express directly the true temperature as a function of the brightness temperature, *i. e.*, the temperature scale for the substance. Thus

$$\frac{1}{T} = \frac{1}{S} + \frac{\lambda}{C_2} ln\epsilon.$$

The relation between T and S for tungsten is given in Table II. by steps of 200°. The highest temperature refers to the melting point of tungsten under atmospheric pressure, further considerations concerning which appear below. The last column of the table indicates the uncertainty in the true temperature of tungsten to be ascribed to an uncertainty of 1 per cent. in the emissive power—that which is considered as probable for the results here presented—when computing the true temperature from brightness temperature and emissive power measurements. It gives at once a method of comparing the scale here obtained with that of others. It is readily seen that Langmuir's scale, when shifted so as to agree as to fundamental characteristics, that is as to the gold point temperature and C_2, and which is based on a constant emissive power of 0.46 agrees with the writer's at 1100° K., but differs from it

TABLE II.

Temperature Relations for Tungsten on Basis of $C_2 = 14350\mu \times$ deg. and $T_{au} = 1336°$ K.

S at $\lambda=0.665\mu$.	ϵ at $\lambda=0.665\mu$.	$T-S$.	ΔT in Case $\frac{\Delta\epsilon}{\epsilon}=+0.01$.
1200	.457	56	−0.7
1400	.451	76	1.0
1600	.446	102	1.3
1800	.440	132	1.7
2000	.434	168	2.2
2200	.428	208	2.7
2400	.422	254	3.3
2600	.416	306	3.9
2800	.410	366	4.6
3000	.403	433	5.4
3176 (melting point)	.398	498	6.2

at 2400° K. (the approximate operating temperature of an ordinary 40-watt vacuum tungsten lamp) and at 3675° K. by 18° and 88° respectively. No similar comparison can readily be made with Pirani and Meyer's scale since they used a very different wave-length, but their result of a constant emissive power of 0.44 at 0.532μ is seen from Fig. 3 to be consistent with the writer's only at a temperature in the neighborhood of 2400° K. However, data obtained by Schackelford[1] on emissive powers in the visible region with the aid of helical filaments of various pitches, by Hulbert[2] both as to changes in emissive power with wave-length and with temperature in the ultra-violet region, and by Weniger and Pfund[3] on the reflecting power of tungsten are in very good agreement with those here presented. It is a point worth emphasizing that the tungsten used by them was in the form of wire which had been drawn as in the common commercial method of preparing tungsten filaments while the writer used the squirted paste filaments. Of the remaining individual emissive power values mentioned in the introduction, only that one given by Coblentz for room temperature, 0.474 at 0.65μ, is in good agreement with those presented here.

THE MELTING POINT OF TUNGSTEN.

The brightness temperature of tungsten at the melting point as recorded in Table II. represents the mean of the four results shown in Table III. Other results on the melting point of tungsten have been summarized by Langmuir and Luckey in their papers. Only the four

[1] Loc. cit.

[2] Jour. Frank. Inst., 182, p. 695, 1916; Astrophys. Jour., 40, p. 149, 1917.

[3] Jour. Frank. Inst., 183, p. 354, 1917.

results mentioned have been included, since in connection with these only are the methods sound and the knowledge definite as to effective wave-lengths used and as to the wave-length to which to ascribe the results. The two methods of determining the brightness temperature at the melting point have been well described by Langmuir. The writer

TABLE III.

Data on Melting Point of Tungsten on Basis of $C_2 = 14350\mu \times$ deg. and $T_{Au} = 1336°$ K

Experimenters.	S at λ=0.665μ.	Observations Made on
Mendenhall & Forsythe[1]	3174° K.	Filament melts.
Langmuir[2]	3187	Filament melts and molten arc terminals.
Worthing[3]	3174	Molten arc terminals.
Luckey[4]	3169	Molten arc terminals.
Av.	3176	

[1] Data were obtained at Nela Research Laboratory in summer of 1914 but results have not been published heretofore.

[2] PHYS. REV., II., 6, p. 152, 1915.

[3] Jour. Franklin Inst., 181, p. 417, 1916. PHYS. REV., II., 7, p. 497, 1916.

[4] PHYS. REV., II., 9, p. 132, 1917.

has been informed by Langmuir that in his measurements on molten arc terminals, the angle of emission varied considerably from the normal. In consequence of the deviation from Lambert's cosine law, higher values are to be expected than if the surface had been viewed normally. However, both Luckey and the writer in their determinations viewed the surfaces normally or nearly so. This might in part explain the high value obtained by Langmuir on the molten arc terminal. However, there remains as unexplained his still higher value from the filament melt data. Considerations of effective wave-lengths brought forth in a subsequent paper by Hyde, Cady and Forsythe[1] together with certain considerations noted above indicate that Langmuir's results on a basis of $C_2 = 14350\mu \times$ deg. should give as an average 3191° K. for S and that this is to be ascribed to 0.661μ. Reducing the results of all so as to refer to 0.665μ has led to the results shown. An equally weighted average has been accepted for the final result. Making use of the emissive power curve here presented, 3674° K. or in round numbers 3675° K. results as the true temperature for the melting point. The uncertainty as to this, granting the fundamental bases of the temperature scale, would seem to be not greater than 15°.

EFFECT ON PREVIOSLY PUBLISHED RESULTS.

The results on thermal and electrical conductivity and Thomson effect previously obtained, expressed in terms of the new temperature scale,

[1] Loc. cit.

are incorporated in Table IV. The radiation intensity values (see also Fig. 4) in reality are the results of measurements at various times on five

TABLE IV.

Previous Data Corrected to New Temperature Scale.

T in °K.	k in $\frac{\text{watts}}{\text{cm.} \times \text{deg.}}$.	$10^{-5} \times \frac{k}{\sqrt{T}}$ in C.G.S. Units.	σ in $\frac{\text{microvolts}}{\text{degree}}$.	E in $\frac{\text{watts}}{\text{cm.}^2}$.	$\frac{T}{E}\frac{dE}{dT}$.
1500	1.01	2.80[1]		5.7	5.21
1700	1.07	3.06		10.8	5.06
1900	1.12	3.29	−20	18.8	4.93
2100	1.17	3.50	−24	30.6	4.81
2300	1.21	3.69	−28	47.2	4.70
2500	1.25	3.87		69.7	4.60
2700	1.29	4.02		98.9	4.50

[1] It is to be noted that the values originally published were in error by the factor 10.

filaments as indicated by the different symbols used in the plat. The results in all cases are free from effects due to cool filament terminals. Three of the lamps possessed very fine potential leads of tungsten wire tied to the larger filaments. The remaining two lamps each possessed

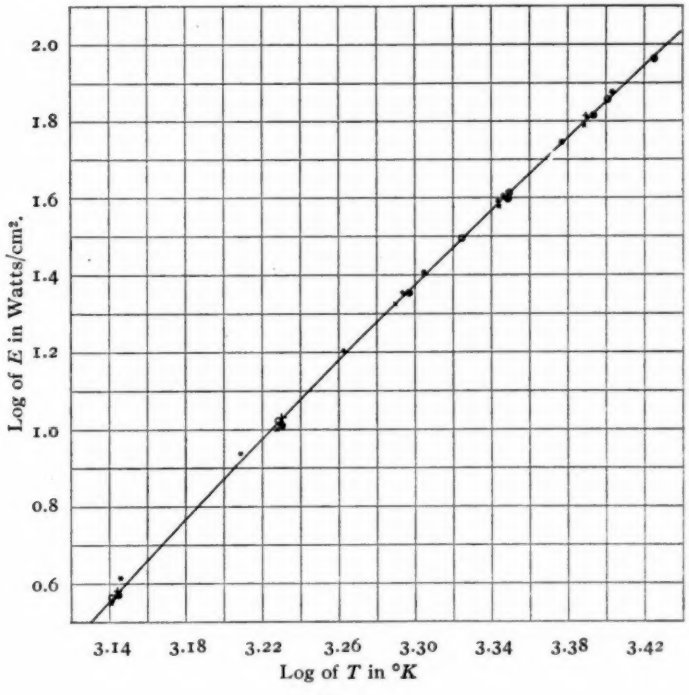

Fig. 4.

Radiation intensity of tungsten as a function of temperature.

two filaments with separate leads which differed only in length, so that by taking differences end effects were eliminated here also. These results may be expressed by the empirical equation

$$\log E = 1.379 + 4.87(\log T - 3.3) - 1.4(\log T - 3.3)^2.$$

It has been assumed that, for practical purposes, the bulbs of the lamps containing these filaments were at negligibly low temperatures. The relative rate of change in emission intensity with relative change in temperature is given under $(T/E)(dE/dT)$ in the table. This quantity for a black body is the exponent 4 occurring in the Stefan-Boltzman equation. The results show a progressive approach toward the black-body radiation in this one respect, but not much significance is to be attached to this since just as fundamental a progressive deviation from black body radiation is shown by the emissive power variation in the visible spectrum.

SUMMARY.

1. A method of determining the emissive power of a substance at incandescent temperatures has been described.

2. A method has been described for determining the wave-lengths to which brightness temperature and emissive power measurements made with the aid of colored glass pyrometer screens are to be ascribed.

3. The emissive power of tungsten at 0.467μ and 0.665μ as a function of temperature have been determined for temperatures up to 3200° K. (Fig. 2 and Table II.).

4. The relation between the true temperature and the brightness temperature at 0.665μ for tungsten has been computed (Table II.).

5. Determinations of the melting point of tungsten have been made. From a consideration of these and other data, 3675° K. ($C_2 = 14350\mu \times$ deg., Tau = 1336° K.) has been obtained as the most probable value for this constant.

6. The radiation intensity as a function of the temperature has been determined for tungsten (Table IV.).

7. Previous data on thermal conductivity and on Thomson effect have been recomputed on the basis of the new temperature scale (Table IV.).

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,
NELA PARK, CLEVELAND, OHIO,
June, 1917.

COLOR TEMPERATURE SCALES FOR TUNGSTEN AND CARBON.

BY E. P. HYDE, F. E. CADY AND W. E. FORSYTHE.

I. INTRODUCTION.

IN a paper published in 1909 by two of the present authors[1] in collaboration with Middlekauff, a new method was proposed and applied for studying the selective radiating properties of certain metals such as tungsten, tantalum and osmium as compared with a black body or with untreated carbon which approximates a black body in its radiation. This method was based on comparative measurements of the ratio of visible to total radiation for the substance to be investigated and for the black body when the two radiating bodies were heated to such arbitrary unknown temperatures that the integral color of the visible radiation was as nearly as possible the same for the two. This " color match " method was further elaborated in subsequent papers,[2] and a more rigorous definition of the " color match " criterion was given.

One of the earliest methods of estimating temperatures in industrial work was based on rough eye observations of the color of the hot body, but this has given place largely to other methods of greater precision based upon different principles. In 1907 Morris, Stroude and Ellis,[3] in a study of the relative operating temperatures of different incandescent lamps, assumed as a starting point equality of temperatures when the color of the light from the various lamps was the same, owing, as they state, to " the great divergence in the figures published by various experimenters for this quantity." In the same year Leder[4] obtained the distribution of energy in the emission of the Hefner lamp by determining the temperature of the black body when it had the same energy distribution in the visible spectrum. But in neither of these investigations was the color match method employed as a means of studying the radiating properties of metals, nor was any consideration given to the significance of the " color temperature " scale, or to its relation to true and black body " brightness temperatures " of incandescent metals.

[1] Trans. Illum. Eng. Soc., *4*, p. 334, 1909. Presented before Am. Phys. Soc., Oct., 1908.
[2] Jour. of Frank. Inst., *169*, p. 439, and *170*, p. 26, 1910; Astrophys. Jour., *36*, p. 89, 1912.
[3] Elec., *59*, p. 584, 1907.
[4] Ann. d. Phys., Ser. 4, *24*, p. 305, 1907.

It has been customary for some years past to give a number to indicate the temperature of any radiating body by determining the temperature of the black body at which the emission intensity in some chosen wave-length is the same as that of the radiating body. This temperature is the "red" or "green" or "blue" "black-body temperature," the exact wave-length being given in accurate work. According to the method discussed in the present paper the comparison with the black body is made on the basis of the same distribution of energy in some limited region of the spectrum, usually in the visible for convenience, rather than on the basis of the same emission intensities. Hence the term "color match temperature." But since in general, precise agreement in energy distribution of the radiation from a black body and of that from one of the radiating metals studied can never be secured, the "color match temperature" has been defined more accurately as that temperature of a black body at which the relative emission intensities in some chosen two wave-lengths are the same as those of the radiating metal under investigation. Numerous experiments showed, however, that the actual match in color with a black body could be so nearly obtained for carbon, tungsten, tantalum, platinum, and osmium that the experimental errors involved in bringing these various substances to a color match by the use of an ordinary Lummer-Brodhun contrast photometer were less than the errors involved in attempting to bring the various substances to the same relative emission intensities in two wave-lengths by the use of a spectrophotometer. Consequently, after establishing this fact for the various metals to be studied the spectrophotometric method was abandoned for the more convenient method of "color match," though for accuracy of conception it must always be borne in mind that the result accomplished consists in the establishment of a condition of equal relative emission intensities in some two wave-lengths near the ends of the visible spectrum—say at 0.5μ and 0.7μ.

As a consequence of the results obtained with this method, and from other knowledge of the radiating properties of these metals it was concluded that quite probably the "black body color temperature" of any of these metals would be higher than the true temperature. If this be true then the color temperature and the "black body temperature" obtained in the customary way, and which we shall hereafter designate[1]

[1] The "color match temperature" is also a black body temperature and so it becomes necessary to designate more precisely the so-called "black body temperature" defined in the customary way. It is proposed, therefore, to designate the latter as "black body brightness temperature," or more briefly, "brightness temperature," giving the wave-length where necessary, and to designate the former as "black body color temperature," or more briefly, "color temperature," giving the two wave-lengths in cases where an integral color match cannot be obtained, or is theoretically insufficient.

the " black body brightness temperature," or, for the sake of brevity, " brightness temperature " would give two limits between which the true temperature would lie, and since the latter is so difficult of measurement, the ascertainment of the upper and lower limits would give valuable information. Moreover the simplicity of the process of determining the color temperature suggested the advisability of using this method to give a number to the temperature of radiating metals instead of the older and more commonly used method of determining the brightness temperature which involves more elaborate apparatus, and, in the case of filaments of small diameter, is subject to possible large errors. If suitable apparatus is available, and proper precautions are taken, the brightness temperature may be obtained with greater accuracy.

In the present experiments the color temperatures of tungsten and carbon are determined, and comparison is made between color temperature, brightness temperature and true temperature of tungsten, using Worthing's[1] data for the latter, and between color temperature and brightness temperature of carbon. The relation is also determined between color temperature and lamp efficiency in lumens per watt, so that it may be possible to locate the color temperature and also the true temperature from measurements of lamp efficiency.

In the earlier papers by two of the authors, to which reference already has been made, the color temperature of tungsten at low voltage was measured directly against a black body, and color temperatures at higher voltages were determined by spectrophotometric comparisons. Moreover comparative data were presented on the brightness temperatures from observations by Waidner and Burgess, but, as will be pointed out later, these early values were only approximate as the emphasis at that time was placed on the application of the " color match " method in the study of selective radiation. In 1915, Shackelford,[2] working in this laboratory, showed that the color of the radiation from the inside of a helical tungsten filament was not so white as that of the radiation from the exterior of the helix, even though the temperature inside was at least as great as that outside. Subsequently Langmuir[3] published the same observation and employed the observed data to given an approximate scale of color temperatures.

Paterson and Dudding[4] in a recent investigation *assumed* that the color temperatures are approximately the same as the true temperatures and obtained results which seemed to show that this assumption was not

[1] Jour. of Frank. Inst., *181*, p. 417, 1916; PHYS. REV., Ser. II.

[2] Jour. of Frank. Inst., *180*, p. 619, 1915; PHYS. REV., Ser. II., *8*, p. 470, 1916.

[3] PHYS. REV., Ser. II., *7*, p. 302, 1916.

[4] Proc. of Phys. Soc. (Lond.), *27*, p. 230, 1915.

greatly in error, as they were not attempting to work to an accuracy greater than 1 per cent. in temperature.

II. APPARATUS AND METHOD.

An outline of the arrangement of the apparatus used to make the measurements is shown in Fig. 1. The furnace shown diagrammatically in Fig. 2 was a vacuum carbon tube furnace somewhat similar to one already described.[1] This furnace, with different graphite tubes, was used

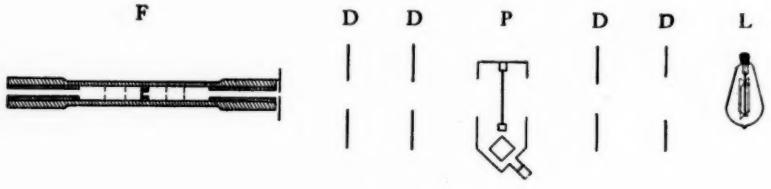

Fig. 1.
Arrangement of apparatus.

in most of the experiments, although some check measurements at low temperatures were made, with platinum-wound porcelain and alundum tube black-body furnaces of the Lummer-Kurlbaum type. The carbon furnace shown in the figure was operated from a transformer supplied with 440 volts which was stepped down to 40 volts. With this source of supply there were required from 50 to 100 amperes through the primary to heat the furnace to temperatures ranging from 1600° K. to 2600° K. The current through the primary could readily be varied in small steps and so the current through the heater tube, and consequently the temperature of the heater tube, were easily controlled.

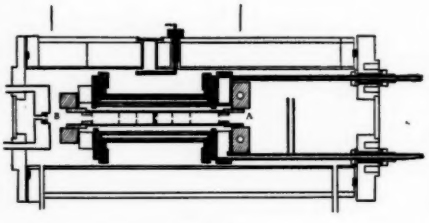

Fig. 2.
Diagrammatic sketch of furnace. A. Heater tube. B. Limiting diaphragm.

Diaphragms, as shown, were very carefully located so that no light reached the photometer except from the central diaphragm. This central diaphragm was made as thin as possible, being only a fraction of a millimeter thick at the central part. All diaphragms were so cut along the outer edge that they touched the heater tube only along two V-shaped edges.

The furnace was so mounted that observations could be made through

[1] Astrophys. Jour. *34*, p. 353, 1911.

the diaphragm at either end. Inasmuch as the heater tube was mounted symmetrically inside the container, and the diaphragms inside the heater tube were equally spaced on each side of the central one, there was no reason to expect differences in temperature or blackness between the two ends.

The temperature of the central diaphragm was measured by means of a laboratory form of the Holborn-Kurlbaum optical pyrometer, directed toward one end of the furnace while a color match was being determined for the light coming through the other end. Thus changes in the temperature due to slight changes in the heating current could be detected and corrected for. In order to be sure that the temperatures at the two sides of the diaphragm were the same, measurements were made on the temperature at each side with different optical pyrometers, before and after each set of measurements on the color. In no case was a larger difference found than 2° or 3° C. The pyrometers were calibrated, using a platinum-wound black-body furnace held at the temperature of melting palladium taken at 1828° K. Extrapolations for the higher temperatures were made by means of Wien's equation using sectored disks. Two thicknesses of red glass (6.8 mm.) (Rotfilter No. F 4512) were used before the eyepiece of the pyrometer. The effective wave-lengths of the red glass were obtained in a previous investigation.[1] The temperature scale[2] used was based on the following values:

Melting point of gold = 1336° K.,

Melting point of palladium = 1828° K.,

$C_2 = 14{,}350\mu \times \text{deg.}$

In making the color temperature determinations the integral light from the furnace was matched in color with that from a comparison lamp using a Lummer-Brodhun contrast photometer and the black body color temperature was transferred to the test lamps by the substitution method.

In making a set of measurements a reading was first taken in the neighborhood of 1800° K., then at higher points and at the end again in the neighborhood of 1800° K. If the first and last readings were in good agreement it was assumed that working conditions were satisfactory. In the early part of this work the only diaphragms in the heater tube were central diaphragms and the diaphragms near the end. While no very great differences were obtained when the heater tube was used as shown, greater weight is given to the later results.

[1] Astrophys. Jour., *42*, p. 294, 1915.

[2] The reasons for the adoption of this temperature scale will be published in the October number of the General Electric Review.

The brightness temperatures of the lamps were determined directly with the same optical pyrometer using a very large magnification. This temperature for the tungsten lamp was also measured by color matching the lamps under investigation with a tungsten lamp having a large filament (0.25 mm. diam.), then measuring the brightness temperature of the large filament. The values obtained by the two methods checked very well but the latter gave the least variations in the result. The values found for the brightness temperature of the tungsten filament as a function of the mean horizontal candles per watt, check well within the limit of error with results on the same relation obtained previously.[1] The values for carbon do not show as good an agreement, due no doubt to the fact that in the previous case treated carbon filaments were used, while in this work untreated filaments were used. It was not possible to get untreated carbon lamps with sufficiently uniform filaments. Consequently the brightness temperatures of the carbon filaments were measured in several places and the mean taken.

The comparison lamp for most of the measurements, and three of the test lamps were 40-watt, 110-volt, drawn-wire, vacuum tungsten lamps. These lamps had their lower supports welded and at the upper supports the filament was held taut by a coiled spring. The other test lamp was a 100-watt, 120-volt tungsten lamp of the old type having welded lower supports, and is the same lamp used in previous investigations.[2] In some check measurements a 25-watt tungsten lamp of the same type as the 40-watt lamp was used, and a 30-watt, 110-volt anchored-oval carbon filament lamp was used as a comparison lamp. In determining the brightness temperatures a single-loop vacuum tungsten lamp was used, made with a filament of 10 mil (0.254 mm. diameter) wire and about 30 cm. long in a bulb approximately 5 inches (12.7 cm.) in diameter.

III. Correction Determinations.

The glass serving as the window of the furnace was not entirely non-selective in its transmission, thus introducing a possible source of error. As at all times the substitution method was employed the error was practically avoided by inserting the same glass between the photometer and the test lamp when transferring from the comparison lamp to the test lamp. Hence this source of error was not directly determined and is not included in the applied corrections (see Table I.).

The selective absorption of the lamp bulbs was corrected by measure-

[1] PHYS. REV., *34*, p. 333, 1912. (Correction must be made to the temperature scale used in the present investigation.)

[2] Jour. of Frank. Inst., *169*, p. 439, 1910.

TABLE I.

Relation between Lumens per Watt and Color Temperature for a Tungsten Lamp.

Lumens per Watt (Uncorrected).	Color Temperature (Uncorrected).	Lumens per Watt (Corrected).	Color Temperature (Corrected).
0.5	1644	0.58	1663
1.0	1777	1.14	1794
1.5	1866	1.70	1883
2.0	1939	2.26	1955
2.5	1998	2.82	2014
3.0	2050	3.37	2066
3.5	2096	3.93	2112
4.0	2138	4.48	2153
4.5	2175	5.02	2190
5.0	2208	5.57	2224
5.5	2241	6.12	2257
6.0	2269	6.66	2285
6.5	2299	7.21	2315
7.0	2327	7.76	2343
7.5	2354	8.30	2370
8.0	2380	8.85	2397
8.5	2406	9.39	2423
9.0	2431	9.94	2449

ments of the transmission for different wave-lengths with a spectrophotometer. This correction expressed in terms of the change in color temperature amounted to about 6° at 1800° K. The lumens per watt of the lamps were corrected for the cooling effect of the leading-in and supporting wires.[1] A correction to the measured lumens was made also for the absorption of the lamp bulbs. These corrections may be evaluated from the data on uncorrected and corrected color temperature and lumens per watt as given in Table I. (which see).

IV. EXPERIMENTAL RESULTS.

(*a*) *Color Temperature versus Lumens per Watt.*—In order to show the agreement among the various observations all of the observed points are plotted in Fig. 3, in which the coördinates are color temperature and voltage of $T - 1$, a 120-volt, 100-watt tungsten vacuum lamp which has been used in previous experiments involving color temperature. It is seen that for the most part the points lie within 5° of the curve, the worst deviation being 16°.

Before presenting the experimental results obtained on the relations between the color temperature, the brightness temperature and the true temperature of tungsten, it seems best to give the observed relation between the color temperature of tungsten and the lamp efficiency,

[1] Trans. Illum. Eng. Soc. (U. S.), *6*, p. 238, 1911.

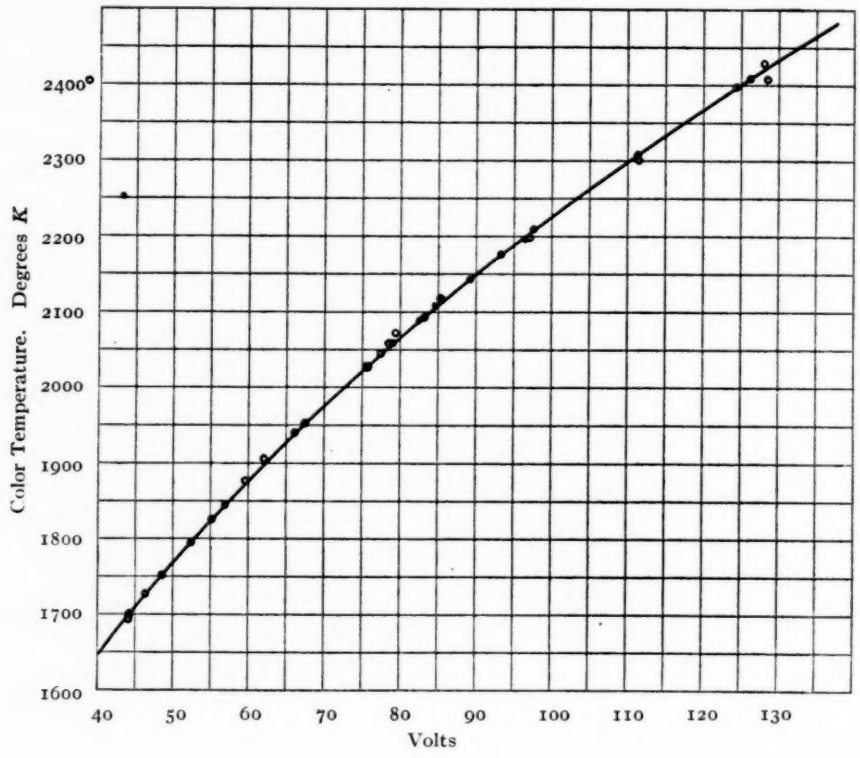

Fig. 3.

The relation between color temperature and volts of a certain tungsten lamp, showing agreement of observed data.

expressed in lumens per watt. The results are shown in Table I. and Fig. 4. This relation is independent of the size and shape of the filament, provided it is operated in a vacuum. The lumens per watt of a lamp are relatively easily determined with a moderately high accuracy, and so by establishing the relation between the lumens per watt and the color temperature the latter may readily be found for any tungsten lamp if the various corrections for bulb absorption and conduction losses are known. Since these latter, though of the same magnitude for different lamps of the same general type, are still somewhat different for different bulbs and for filaments of different size, the results are given in the original uncorrected values, and also corrected for all these sources of error. Any investigator may determine the magnitude of these correction factors for any lamp he may study, or, if so great an accuracy is not required, he may assume that the corrections are sensibly the same for his lamps as for those used in the present investigation, and he may therefore use the uncorrected values.

In the experiments described, four tungsten lamps were carefully color-matched with each other at various points throughout the range. It was found that a definite change in color temperature corresponded to the same relative change in voltage for each lamp within the errors of

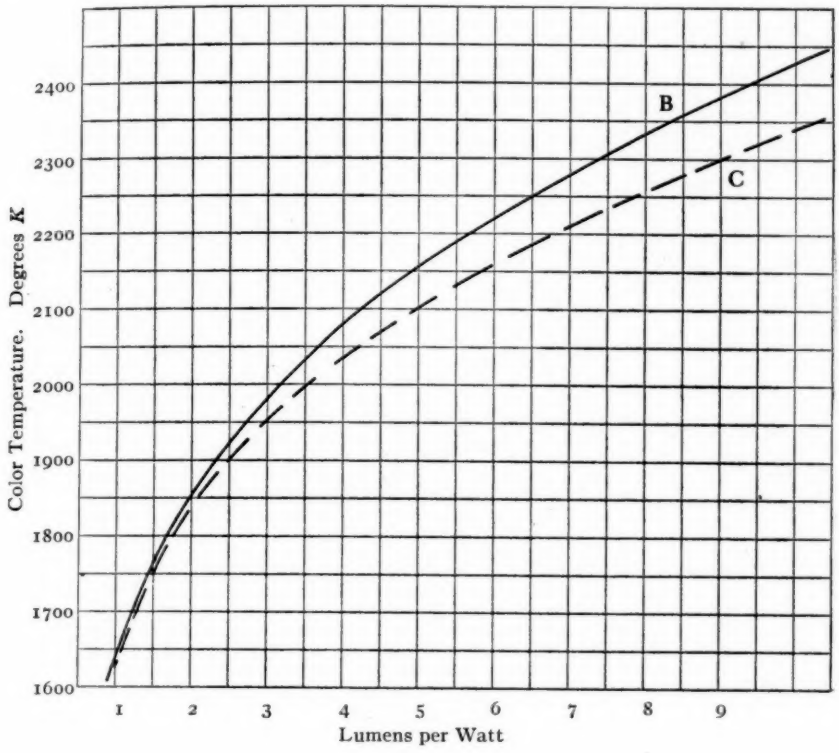

Fig. 4.

The relation between color temperature and lamp efficiency of tungsten. *B.* Author's data. *C.* Paterson and Dudding's data.

observation. Measurements of the mean horizontal candle-power were then made with the lamps operating at voltages to give the same color as that of the carbon standards in terms of which they were being measured. In order to avoid the difficulties due to heterochromatic photometry it was decided to determine the candle-power at other voltages by using the table published by Middlekauff and Skogland.[1]

In Fig. 4, for comparison, the corresponding uncorrected curve for tungsten obtained by Patterson and Dudding is drawn as a dashed line. In the neighborhood of 1600°–1700° K. the two curves are in as close agreement as could be expected in view of the disclaimer of Paterson

[1] Bull. Bureau of Standards, *11*, p. 483, 1915.

and Dudding of an accuracy better than about 20°, but at the higher temperatures the differences between the two curves are much too large to be accounted for on this basis, as at 2400° K. the difference amounts to approximately 85°. In the neighborhood of 2023° K. (1750° C.) the difference is 30°–40° and if this correction were assumed in determining from Paterson and Dudding's data the color temperature of platinum at its melting point, using the tungsten comparison lamp, the melting point would have a color temperature of about 2080° K. (1807° C.) or some 50°–60° above the true temperature, which difference would show beyond question that the color temperature of melting platinum is definitely higher than the true temperature, a result which must follow from other published data on platinum. It may be stated in passing that no attempt will be made to justify the discrepancy between the value found by Paterson and Dudding using a tungsten comparison lamp, and that

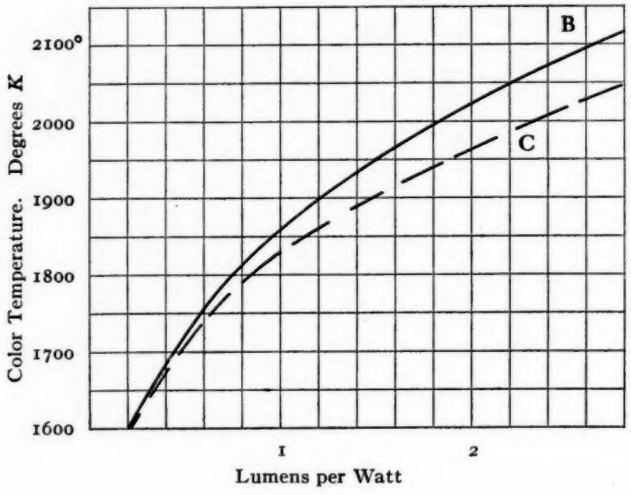

Fig. 5.

The relation between color temperature and lamp efficiency of carbon. *B.* Author's data. *C.* Paterson and Dudding's data.

derived from a similar comparison against a carbon lamp. They should, of course, be the same, but it should be stated once more that these authors do not claim any greater accuracy than that indicated by the difference in results in these measurements.

The data on the relation between color temperature and lumens per watt for untreated carbon are given in their observed, or uncorrected form, in Fig. 5. The corresponding curve obtained by Paterson and Dudding is also given for comparison, but it should be noted that the latter curve is based on measurements on both treated and untreated

carbon filaments, whereas the authors' data are entirely confined to untreated carbon. Owing to the irregularities in the untreated carbon filaments the accuracy attainable is not nearly so great as that possible with tungsten, and this fact, together with the consideration of the relatively smaller errors introduced by end conduction losses, has suggested the inadvisability of attempting to give a corrected curve, as was done for tungsten.

(*b*) *Color Temperature, Brightness Temperature and True Temperature.* —Reference has already been made to the fact that two of the present authors[1] together with Middlekauff some years ago published results on the color and brightness temperatures of various metals, giving also the lumens per watt, but those results will be found somewhat different from the present values owing to several reasons, principally the lack of

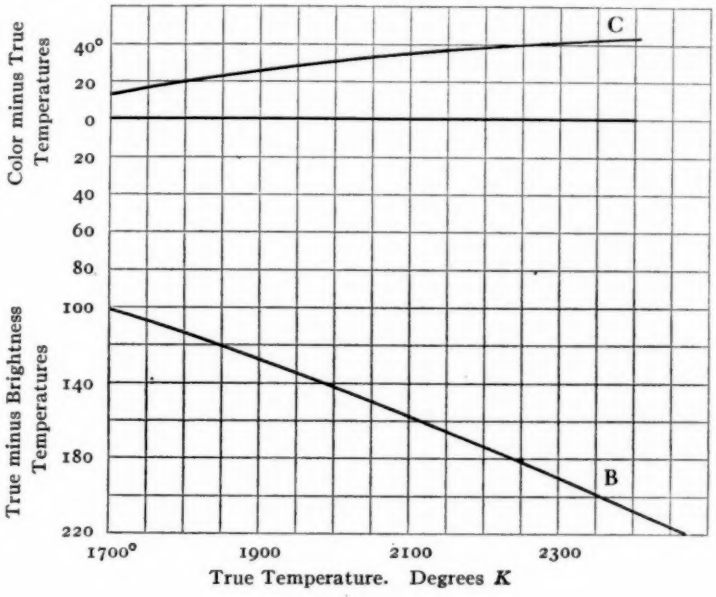

Fig. 6.

The relation between color, true, and brightness temperatures of tungsten.

knowledge at that time of various correction factors to be applied. Thus the values of color temperature given were for the most part determined from energy-distribution curves obtained by means of the spectrophotometer, and the slit-width corrections in spectrophotometric measurements had not then been investigated.[2] Moreover the brightness temperatures given in the earlier publication were intended only as approximate values, as was stated in the paper.

[1] See references 1 and 2, p. 395.

[2] Astrophys. Jour., *35*, p. 237, 1912.

The results obtained in the present investigation are shown in Figs. 6 and 7. In Fig. 6 the differences between the true temperatures[1] and the color and brightness temperatures for tungsten are given, the color temperature values being corrected for the various errors enumerated in an earlier section. The observations bore out the expectation that the color temperature would be larger than the true temperature for all temperatures, whereas the brightness temperatures are of course con-

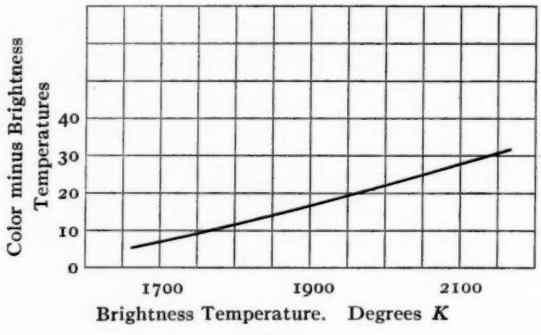

Fig. 7.

The relation between color and brightness temperatures of carbon.

siderably smaller. The brightness and color temperatures are seen to give upper and lower limits to the true temperature, with the difference from the true temperature considerably less in the case of the color temperature. This general result would very probably be found to obtain for a number of metals, since available data on the reflecting powers and also other indications of the optical properties of many metals show quite general selective absorption of the shorter wave-lengths.

It should be noted that the difference curve between color temperature and true temperature, if prolonged toward lower temperatures, would apparently cross the axis, indicating that the color temperature would be lower than the true temperature if the comparison were made at sufficiently low temperatures. There is no physical reason to believe this to be true. It is far more probable that the shape of the difference curve is subject to modification. In the first place it must be emphasized that experimental errors may occur as large as 5° and possibly somewhat larger. In the second place differences in the emissivities of different samples of tungsten may modify this difference curve slightly, since the true temperature and the color temperature are obtained independently from the brightness temperature, using different lamps.

Direct measurements of the relative intensity of emission in the red

[1] Jour. of Frank. Inst., *181*, p. 417, 1916; PHYS. REV., Ser. II.

and in the blue, with an optical pyrometer, calibrated in brightness at the two wave-lengths by comparison with a black body, give a color temperature curve which differs slightly from the color temperature curve obtained by integral color match, and indicate a difference curve between the color temperature and the true temperature more nearly of the form one might expect. But since the difference between the curves is quite probably within the experimental accuracy, and since, moreover, there is always the possibility that with so selective a radiating body as tungsten the integral color temperature may differ slightly over a wide range of temperature from the color temperature determined from any two chosen wave-lengths (approximately 0.665μ and 0.467μ in these experiments), it has seemed advisable to adhere to the observed curve as given in Fig. 6, since this is the curve of more practical value.

Independent measurements with a spectrophotometer, and computations from observed data on the brightness of a black body (to be published shortly) both give color temperature scales in substantial agreement with the observed scale (Fig. 6).

The results for carbon are given in Fig. 7. As stated previously the data on carbon do not justify any attempt to apply corrections similar to those determined for tungsten. In the case of carbon, since there are no reliable data on the true temperature, the differences between the brightness temperatures and the color temperatures only are plotted.

(*c*) *Relation between Color Temperature and Watts.*—Various attempts have been made to determine the exponent β in the assumed generalized form of the Stefan-Boltzmann law for metals,

$$E = \sigma T^{\beta}.$$

It is of interest to inquire into the possible existence of a similar relationship between the total radiation and the color temperature. If the color temperature of a lamp is known at some one wattage and if a simple law is found to hold for the relationship between color temperature and wattage, the application of this law affords a convenient way to establish the entire color temperature scale. The investigation of this relationship is interesting also because its consideration in conjunction with that of other established relationships furnishes a check of the original observed color temperature scale.

Paterson and Dudding give for the above relationship for carbon lamps (including both untreated and flashed filaments)

$$W \alpha T^{4.58},$$

and for tungsten lamps

$$W \alpha T^{5.1},$$

indicating a constant exponent for both metals.

Measurements by the present authors, of watts *vs.* color temperature (uncorrected) for tungsten and carbon, with the corresponding computed values of the exponent β at different regions of the total temperature interval investigated are given in Tables II. and III.

Considering the data for tungsten (Table II.) it is seen that there is a

TABLE II.

Relation between Color Temperature and Relative Watts for a Tungsten Lamp Including Corresponding Values of β.

Color Temperature (Uncorrected).	Relative Watts.[1]	β Computed from Data on Color Temperature and Watts.	β Computed from Other Data.
1750	22.3	4.99	4.93
1800	25.7	4.98	4.90
1850	29.3	4.79	4.87
1900	33.4	4.89	4.84
1950	37.8	4.80	4.81
2000	42.6	4.77	4.79
2050	47.9	4.68	4.76
2100	53.6	4.69	4.74
2150	60.0	4.82	4.72
2200	66.9	4.73	4.70
2250	74.5	4.79	4.68
2300	82.6	4.72	4.67
2350	91.5	4.71	4.65
2400	100.9	4.66	4.64
2450	110.2	4.26	4.62
Average		4.75	4.75

[1] 100 = watts corresponding to 1.2 watts per mean horizontal candle.

TABLE III.

Relation between Color Temperature and Relative Watts for an Untreated Carbon Lamp Including Values of β.

Color Temperature (Uncorrected).	Relative Watts.[1]	β.
1650	38.9	——
1700	43.8	4.03
1750	49.2	4.01
1800	55.1	4.06
1850	61.5	3.95
1900	68.4	3.97
1950	76.0	4.11
2000	84.2	4.00
2050	92.9	3.97
2100	102.4	4.09
Average		4.02

[1] 100 = watts corresponding to 4 watts per mean horizontal candle.

distinct indication of a gradual decrease in β as the temperature increases, although the successive values of β are not always consistent. This inconsistency is due to slight irregularities in the observed color temperature scale. The average value of β over the observed range of temperature is 4.75, differing from the value of Paterson and Dudding in the direction to be expected in view of the difference between the two temperature scales. The following considerations show, however, that the observed indication of a decreasing β with increasing temperature is verified in fact.

If I represents candle-power, and if T_1 and T_2 are two color temperatures at any region of the interval but differing from each other by an infinitesimal amount, then the following relations hold:

$$(1) \qquad \frac{I_2}{I_1} = \left(\frac{T_2}{T_1}\right)^l,$$

$$(2) \qquad \frac{W_2}{W_1} = \left(\frac{T_2}{T_1}\right)^\beta,$$

$$(3) \qquad \frac{I_2}{I_1} = \left(\frac{W_2}{W_1}\right)^{k'} = \left(\frac{T_2}{T_1}\right)^{\beta k'}.$$

From equations (1) and (3)

$$l = \beta k'.$$

If the subscript " o " is used to refer to a black body at the same color temperature, then

$$\frac{l}{l_0} = \frac{\beta k'}{\beta_0 k_0'},$$

and hence, since $\beta_0 = 4$,

$$(4) \qquad \beta = 4 \frac{l}{l_0} \frac{k_0'}{k'}.$$

Now k', which is the percentage change in candle-power for one per cent. change in watts is accurately known for tungsten lamps, uncorrected for end effects, and the corresponding quantity k_0' for a black body is known by computation[1] to within a very small uncertainty. There is, of course, the error in locating the precise color temperature of tungsten,—the very error which gives rise to the present uncertainty as to the constancy of β for tungsten, but the observed color temperature scale is certainly correct to within an error of the order of magnitude of 10° and an error of this magnitude would affect k' by only 0.6 per cent. to 0.7 per cent., whereas the ratio k_0'/k' varies by 5 per cent. over the temperature interval 1700° to 2150° K.

[1] Astrophys. Jour., *36*, p. 89, 1912.

If now l/l_0 can be determined, substitution in equation (4) will give values of β for tungsten. "l" is the exponent giving the relative change of candle-power of tungsten for a small change in color temperature, and "l_0" is the corresponding exponent for a black body. In a previous paper[1] by one of the authors a criterion (Criterion I.) was established for determining the constancy in emissivity of metals in the visible spectrum, and it was shown that if the emissivity in the visible is constant for some metal over a given interval of color temperature, then the relative candle-power of the metal and the black body over that interval of color temperature will be the same, and so for small steps in that interval $l/l_0 = 1$.

Although this criterion was apparently fulfilled for carbon and tantalum, there was found a slight deviation in the case of tungsten, which since that time has been verified and more accurately evaluated.[2] The deviation in the case of tungsten is approximately 1.5 per cent. and sensibly the same over the temperature interval investigated. Putting for l/l_0 its value 0.985 and for k_0'/k' the values obtained as indicated, the values of β may be computed to a fairly high accuracy. Values obtained in this way are given in the fourth column of Table II. The agreement between these values of β and the observed values, given in the third column, is as good as might be expected, and if, reversing the process, a color temperature scale should be constructed from the computed β's it would agree with the observed scale within less than 5°, which is within the experimental error. The average values of β by the two methods are in excellent agreement.

The values of β for untreated carbon are given in Table III. It is probable that β is very nearly, if not quite constant for carbon over the observed temperature interval, and the uncertainty in the observations, owing to the lack of uniformity in the untreated carbon filaments vitiates any effort to analyze the results further. As in the case of tungsten, the average value of β for carbon is less than the value found by Paterson and Dudding, but here again this is to be expected in view of the difference in the corresponding color temperature scales.

SUMMARY.

The "black body color temperatures" or simply the "color temperatures" for tungsten and untreated carbon lamps are given from direct observations against a carbon-tube black-body furnace, plotted against lumens per watt of the lamp.

For tungsten the differences between the "color temperature," the

[1] Loc. cit.

[2] Worthing, loc. cit.

" brightness temperature " (ordinarily called heretofore the " black-body temperature "), and the true temperature are given, and it is pointed out that the color temperature is greater than the true temperature, whereas the brightness temperature, as is well known, is less than the true temperature. The color temperature, however, is much nearer the true temperature.

For carbon the difference between the color temperature and the brightness temperature is given, the true temperature being unknown.

The relation between the color temperature and the watts is investigated, and found, within observational errors to obey approximately an exponential function. It is shown that for tungsten the exponent cannot be a constant, but must decrease slightly from to low high temperatures.

NEW BOOKS.

Radiodynamics. By B. F. MIESSNER. New York: D. Van Nostrand Company, 1916. Pp. v + 206. Price, $2.00.

With the entry of the United States into the war general interest in the subject matter treated in "Radiodynamics" by B. F. Miessner has greatly increased. In the opinion of the reviewer the book should be in the library of any physicist who is at present interesting himself in the problems which our entry into the war has brought to the fore. The general subject treated is communication by means of radiations and control of moving mechanisms by the same means. There is no attempt to explain principles which can of course be studied quite easily from other sources and which are well known to the readers of the REVIEW.

The value of the book lies in its being an up-to-date statement of what has been accomplished and the means employed in the control of mechanisms by means of the various forms of radiant energy.

A. T.

Telephone Apparatus. By GEORGE D. SHEPARDSON. New York: D. Appleton and Co., 1917. Pp. xvii + 337.

This book was written to supply the need for a systematic historical and theoretical treatment of the subject. It will be found useful not only to the telephone engineer, but to the physicist who wishes to keep in touch with the applications of his science. Part I. reviews the fundamental acoustical principles and describes the various types of transmitter and receiver. Part II. discusses signalling equipment. Part III. treats of sources of electromotive force, the uses of condensers and induction coils, and protective devices. The scope of the book does not include the consideration of telephone circuits or of wireless telephony. The descriptions of some of the many devices referred to are necessarily somewhat meager, but there are abundant references to other books and to periodical articles. Several appendixes are devoted to the laws of the magnetic circuit and to a more extended development of the mathematical relations employed in the text. The book is well printed and illustrated, and is provided with good author and subject indexes.

E. P. L.

Atoms. By JEAN PERRIN and translated by D. L. HAMMICK. New York: D. Van Nostrand Co., 1916. Pp. xiv + 211. Price, $2.50.

This translation of the fourth revised edition of this little book will be welcomed by all English-speaking physicists and chemists. The topics treated are: Chemistry and the atomic theory, including the phenomena of solution;

molecular agitation and kinetic theory; the Brownian movement, including a detailed account of the author's work and of Einstein's theory; phenomena due to fluctuations of density in liquids and gases; the radiation of black bodies, Planck's theory of quanta, and Einstein's theory of specific heat: the atom of electricity and the methods of measuring the elementary charge; the genesis and destruction of atoms as manifested in radioactive phenomena. It is to be regretted that no account is given of spectroscopic phenomena which are closely related to atomic theories, such as the Zeeman and Stark effects, and X-ray spectra, or of the work on atomic numbers, or of the theories of atomic structure which have recently attracted so much attention. In spite of these omissions, no other single book known to the reviewer gives such a comprehensive survey of this important field. Although necessarily concise, the treatment is by no means superficial.

An erroneous reference on page xiv to the Rice Institute as the University of Houston has not been corrected by the translator. On page 22 the numbers 653 and 65.3 should be 65.7. On page 49 "*1 cent-milliardième*" is translated "the hundred-thousandth."

E. P. L.

Radioactivity. By FRANCIS P. VENABLE. New York: D. C. Heath and Co., 1917. Pp. vi+54. Price $.50.

This little book is the outcome of work which originally was given in lecture form to students of elementary chemistry. It should prove useful to students of science who have not the time or opportunity for reading the more complete treatises on the subject. While the book is very small, and can be read at a single sitting, yet it contains in a well-arranged and clearly written form the most important results of the work done in this field. The text is unusually free, for so popular a treatment, from errors and misleading statements. Teachers of physics and chemistry will find it a good book for reference for their beginning students.

O. M. S.

X-Rays. By G. W. C. KAYE. Second Edition. London: Longmans Green and Co., 1917. Pp. xxi+285. Price, $3.00 net.

The author does not claim to have written "a treatise or hand-book on X-rays." The book contains brief accounts of some (not all) of the important papers that have been published on X-rays up to the middle of 1916. The second edition does not differ very much from the first. Unfortunately a great deal of material that is now known to be inaccurate, and even incorrect, is described in considerable detail, whereas some of the most important researches are either not mentioned at all, or referred to in a few words. In spite of this the book may be useful to the student of X-rays in that it provides a concise statement (with references) of some of the research work that has been done on X-radiation.

W. D.

The Theory of Measurements. By LUCIUS TUTTLE. Philadelphia: Jefferson Laboratory of Physics, 1916. Pp. xiv+303. Price, $1.25.

Written in the form of a laboratory manual with extended discussions, questions and problems, this book is an elementary treatise on the theory of measurements and computations made from experimentally obtained quantities. The author discusses general considerations of experimental procedure and measurements, the theory of error, accuracy, adjustment of observations, graphical methods, use of slide rule, logarithms, etc. The mathematical knowledge required is no greater than that usually required of a student in a college course of general physics. Each step is introduced by an appropriate laboratory exercise which offers a definite objective illustration of the matter under discussion. This plan of procedure will in no way hinder the use of the book by a reader who can not perform the suggested experiments. In fact it probably adds to the clearness of presentation and improves the treatment for the average reader.

Nowadays it is the tendency to point our laboratory work towards a great emphasis of the *principles* of physics with the probable result that we overlook often the theory of the means which we employ, the theory of quantitative observation and computation. Hence a book like this can well be used to supplement regular courses in laboratory physics. Part of the material found here is in some laboratory manuals, but in general with a treatment very incomplete. It is a serious question whether we have not gone entirely too far in reducing the amount of this kind of work in our elementary laboratory courses. Certainly this book contains much that every advanced student of quantitative experimental science should be thoroughly familiar with.

O. M. S.

Physical Laboratory Experiments for Engineering Students. By SAMUEL SHELDON AND ERICH HAUSMANN. New York: D. Van Nostrand Co., 1917. Pp. v+134.

This book was prepared for the use of sophomore students in the Polytechnic Institute of Brooklyn. Part I. contains thirty experiments on mechanics, sound, heat and light, forty illustrations and diagrams, and an appendix in which are given tables of physical constants. The book is especially adapted for a laboratory course for students who have had some previous knowledge of physics and mathematics including the calculus, but who do not intend to take an advanced laboratory course in physics. The illustrations are of modern apparatus manufactured by well-known makers.

E. J.